Fredholm theory and localization on metric measure spaces

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Let $\mathcal{B}$ be a (unital) commutative Banach algebra and Ω the set of non-trivial multiplicative linear functionals $\omega : \mathcal{B} \to \mathbb{C}$. Gelfand theory tells us that the kernels of these functionals are exactly the maximal ideals of $\mathcal{B}$ and, as a consequence, an element $b \in \mathcal{B}$ is invertible if and only if $\omega(b) \neq 0$ for all $\omega \in \Omega$. A generalization to non-commutative Banach algebras is the *local principle* of Allan and Douglas, also known as *central localization*: Let $\mathcal{B}$ be a Banach algebra, Z a closed subalgebra of the center of $\mathcal{B}$ and Ω the set of maximal ideals of Z. For every $\omega \in \Omega$ let $\mathcal{I}_\omega$ be the smallest ideal of $\mathcal{B}$ which contains ω. Then $b \in \mathcal{B}$ is invertible if and only if $b + \mathcal{I}_\omega$ is invertible in $\mathcal{B}/\mathcal{I}_\omega$ for every $\omega \in \Omega$.

From an operator theory point of view, one of the most important features of the local principle is the application to Calkin algebras. In that case the invertible elements of $\mathcal{B}$ are called Fredholm operators. Therefore, by taking suitable subalgebras, we obtain a characterization of Fredholm operators. However, the central localization is often not sufficient to provide a satisfactory characterization. In this talk we therefore consider a generalization where the ideals $\mathcal{I}_\omega$ do not originate from the center of the algebra. More precisely, we will consider L^p-spaces over metric measure spaces and, under suitable assumptions, characterize Fredholmness in terms of limit operators.

Based on joint work with Christian Seifert.