Volcanic Plumbing

References:
Encyclopedia of Volcanology: pp. 219-248

Categories of Flow

- Laminar
  - Well ordered and smooth
- Turbulent
  - Chaotic irregular motion superimposed
- Reynolds number of ~2000 separates them

Pipe Flow

- Confined flow under steady conditions
- Can be used to model dikes and conduits
- Flow velocity is maximum at the center
- Flow velocity is zero at the boundary

Dimensionless Parameters

- Reynolds number
  - Measures the ratio of inertia to viscous force

    $R_e = \rho LV/\mu$
- Froude number
  - Measures the ratio of inertia to gravity force

    $F_r = V^2/gL$

Conservation of Mass

- Constraints
  - A single pipe
  - Steady flow
- Conservation of mass

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 = Q$$

Where:

Q is the mass flow rate

V is velocity

ρ is density

Newton's Second Law

$\mathbf{F} = d\mathbf{M}/dt$

M is the total linear momentum (mV)

- For fluids crossing a control surface

  $\Sigma \mathbf{F_x} = dm/dt(V_{x2} - V_{x1})$

  dm/dt is the mass flux rate

Conservation of Energy

$Q - W = \Delta E$

- Q is work added to the system
- W is work done by the system

  ΔE is the change in energy of the sys

  $E = U + 1/2mV^2 + mgz$

  Energy = internal + kinetic + potential
- This is the basis for the Bernoulli Equation for frictionless incompressible flow

Bernoulli's Equation

- Application of the 1st law of thermodynamics to steady, non-compressible, non-viscous flow.

  $V^2/2 + pv + gz = \text{constant}$

- Dividing by g, expressions have the units of height

  $V^2/2g + p/\gamma + z = \text{constant}$

  where: v = volume

  $\gamma = g/v$

  z is elevation

Laminar Flow

- Flow between parallel plates
  - Models dike flow
- Flow in a pipe
  - Models conduit flow

Flow Between Parallel Plates

$u = 1/2\mu \; dp/dx \; (y^2 - h^2)$

u is velocity

μ is viscosity

y is distance from center of flow

h is radius of conduit

Poiseuille flow

- Fully developed laminar flow in a pipe.

  $u = 1/4\mu \; dp/dx \; (r^2 - R^2)$

- Applies to magmas rising slowly in a conduit before major exsolution of volatiles

Turbulent Flow

- A power law applies,

  $u/u_{max} = (y/R)^{1/n}$

where y is the distance from the pipe wall to the center.

$1/n = f(Re)$ and ranges from 1/6 to 1/10 for Re from about 10^3 to 10^6

Mean Velocity

- Its value depends on the velocity profile.

U x R = Σ $u$dr

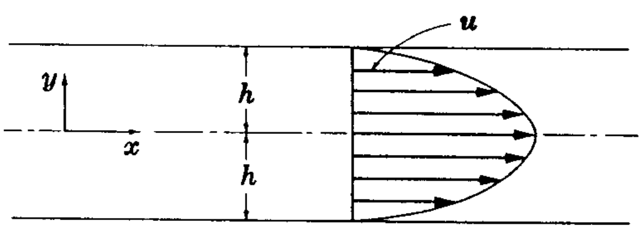
U = mean velocity
R = channel radius
u = local velocity

Head Loss

- There is no way to determine head loss (H) for turbulent flow by purely analytical methods.

H = L/D (v^2 /2g) f
where f is a friction factor

- For laminar flow f ~ $64/R_e$



Fully developed laminar flow between parallel walls.

$$u = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - h^2)$$