The Bisection Method

Suppose that a continuous function f takes on the value of 0 for some values $x_1, x_2, x_3, \ldots, x_n$. If f is polynomial of first or second degree, then finding these values of x is relatively simple. However, f may often be a more complicated function than a simple first or second degree polynomial. If this is the case, then numerical methods must be used to find the values that make $f = 0$.

The first method that will be examined is called the *Bisection Method*. If we know that a root of f lies in the interval $[a, b]$, then it is possilbe to shorten the interval (by cutting it in half) until a close approximation of one of the roots of f is determined. This is the fundamental idea behind the Bisection Method.

In order for this method to work, $f(a)$ and $f(b)$ must have opposite signs, i.e. one value is negative, while the other is positive. Since f is continuous, we are guaranteed by Bolzano's Intermediate Value Theorem that f takes on the value 0 given that the two values of f, $f(a)$ and $f(b)$ are opposite in sign. Consider a point $c \in [a, b]$ such that the distance from c to a is exactly the same as the distance from c to b. Now c is either a zero of f or it is not. If c is a zero of f, then we have found one of our roots. If c is not a zero of f, then either the set $\{f(a), f(c)\}$ or $\{f(b), f(c)\}$ contains two elements that opposite signs. If we choose the set that contains the opposite signs, then we can bisect the interval again repeatedly until we find the desired root within some tolerance ϵ.