Newton's Method

The Bisection Method is a successful method for determining the location of a root of some function f. However, the Bisection Method has the disadvantage of converging to a solution rather slowly. Newton's Method is an improved method for finding the roots of a function since it is based on the geometric idea of successively approximating a curve by tagent lines.

Let f be a differentiable function that has a root at some point r on the x-axis. Let x_1 be an initial estimate for the value of r. Now from Calculus, we know that the equation of the of the line tangent to the graph at the point $(x_1, f(x_1))$ is:

$$y = f(x_1) + f'(x_1)(x - x_1).$$

This line crosses the x-axis at the point:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}.$$

If x_1 is replaced by the second estimate, x_2, then the point x_3 is obtained, and so on. Thus in general, x_{n+1} can be obtained from the point x_n by the formula,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

This process is iteratively calculated until $f(x_n)$ falls below a certain tolerance given by the value of ϵ, i.e. $|f(x_n)| \leq \epsilon$. The point at which this occurs, x_n, is given as an approximation of the one of the roots of f.