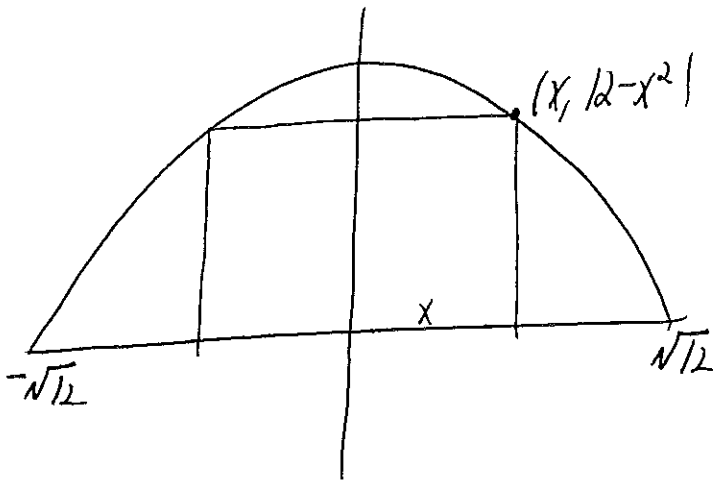


# SOLUTIONS

Math 141A- Midterm Exam #3 - November 17, 2014

1. (20 points) A rectangle has its base on the x axis and its upper two vertices on the parabola  $y = 12 - x^2$ . What is the largest area the rectangle can have and what are its dimensions?



$$\begin{aligned} \text{Area} = A(x) &= 2x(12 - x^2) \\ &= 24x - 2x^3 \end{aligned} \quad 0 \leq x \leq \sqrt{12}$$

$$A'(x) = 24 - 6x^2$$

$$\text{set} = 0$$

$$24 - 6x^2 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

x	A(x)
0	0
2	32
$\sqrt{12}$	0

Max area = 32

Dimensions are 4x8

2. (10 points) Suppose  $f'(x) = \frac{1}{x} + \sin x$ , for  $x > 0$ . Suppose also that  $f(1) = 2$ . Find  $f(x)$ .

$$f(x) = \ln x - \cos x + C$$

$$f(1) = 2 = \ln 1 - \cos 1 + C$$

$$2 = 0 - \cos 1 + C$$

$$C = 2 + \cos 1$$

$$f(x) = \ln x - \cos x + 2 + \cos 1$$

3. (10 points) Find

$$\lim_{x \rightarrow 1^+} x^{1/(1-x)}$$

type  $1^\infty$

$$y = x^{1/(1-x)}$$

$$\ln y = \frac{\ln x}{1-x}$$

$$\lim_{x \rightarrow 1^+} \ln y = \lim_{x \rightarrow 1^+} \frac{\ln x}{1-x} \quad \text{type } \frac{0}{0} \text{ use LHR}$$

$$\stackrel{\text{LHR}}{=} \lim_{x \rightarrow 1^+} \frac{1/x}{-1} = -1$$

$$\text{So } \lim_{x \rightarrow 1^+} \ln y = -1 \Rightarrow \lim_{x \rightarrow 1^+} y = e^{-1}$$

$$= \left( \frac{1}{e} \right)$$

4. (15 points) Let  $f(x) = x^4 - 4x^3 + 10$ . Find the critical values and classify them as local max, min, or neither using the second derivative test. If the second derivative test fails, you may use the first derivative test.

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$$

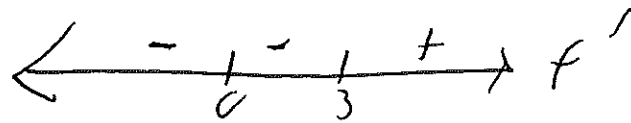
crit points  $x=0, 3$

$$f'' = 12x^2 - 24x$$

$$f''(0) = 0 \quad \text{no info}$$

$$f''(3) = 46 > 0 \quad \text{so } \text{local min}$$

Use 1<sup>st</sup> der test for  $x=0$



so Neither max nor min

5. (20 points) Let

$$f(x) = \frac{(x+1)^2}{1+x^2}$$

Using the quotient rule we obtain:

$$f'(x) = \frac{2(1-x^2)}{(1+x^2)^2}, \quad f''(x) = \frac{4x(x^2-3)}{(1+x^2)^3}$$

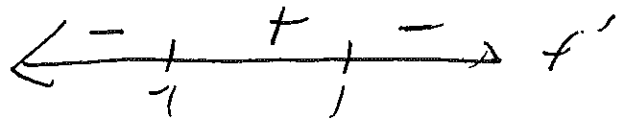
- Find all  $x$  and  $y$  intercepts and any asymptotes.
- Find the intervals where  $f(x)$  is increasing or decreasing and any local maximums or local minimums.
- Find the intervals where  $f(x)$  is concave up or concave down, and determine any inflection points.
- Neatly sketch the graph of  $y = f(x)$ , Label the  $x$  and  $y$  coordinates of any intercepts, local extrema and inflection points.

a.  $\boxed{(-1, 0) \quad (0, 1)}$

$\lim_{x \rightarrow \pm\infty} f(x) = 1$

so  $\boxed{\text{horiz asym } y=1}$

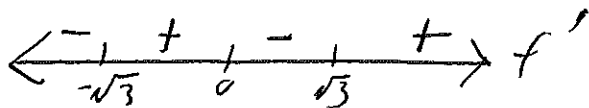
b  $f' = 0$  at  $x = \pm 1$



$\boxed{\begin{array}{l} \text{Decreasing } (-\infty, -1) \cup (1, \infty) \\ \text{Increasing } (-1, 1) \end{array}}$

$\boxed{\begin{array}{l} \text{local min } (-1, 0) \\ \text{local max } (1, 2) \end{array}}$

c.  $f''=0$  at  $0, \sqrt{3}, -\sqrt{3}$



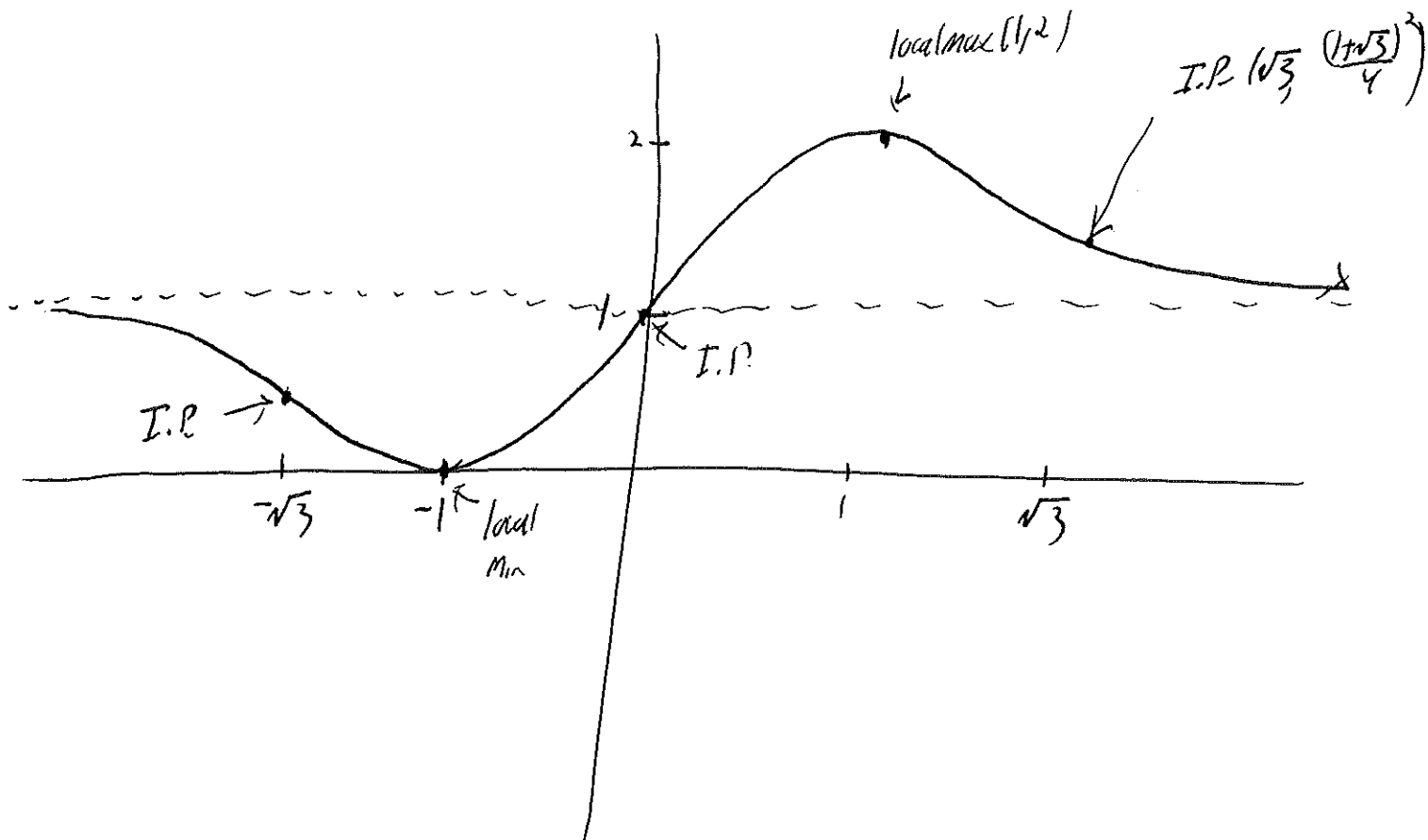
concave down  $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$   
 concave up  $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

$$f(\sqrt{3}) = \frac{(1+\sqrt{3})^2}{4}$$

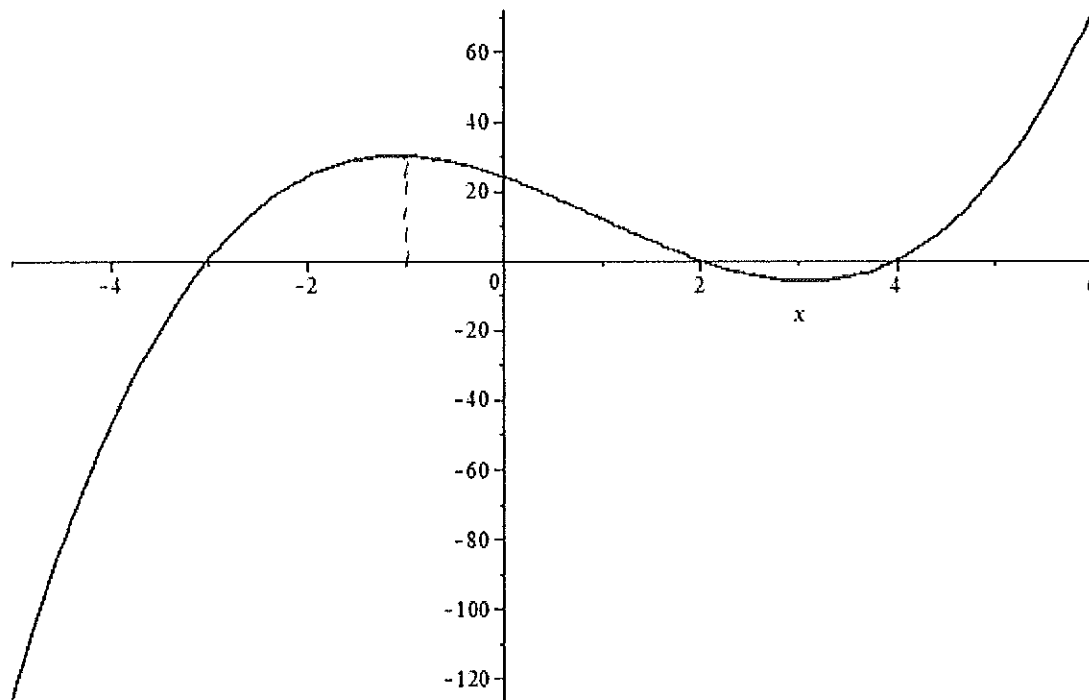
$$f(-\sqrt{3}) = \frac{(1-\sqrt{3})^2}{4}$$

I.P.  $(\sqrt{3}, \frac{(1+\sqrt{3})^3}{4})$   
 $(-\sqrt{3}, \frac{(1-\sqrt{3})^3}{4})$   
 $(0, 1)$

d.



6. (15 points) Below is the graph of  $y = f'(x)$ . Find the intervals where the original function  $f(x)$  is increasing/decreasing and concave up/down.



increasing  $(-3, 2) \cup (4, \infty)$  (since  $f' \geq 0$ )

decreasing  $(-\infty, -3) \cup (2, 4)$  " "  $\leq 0$

concave up where  $f'' > 0$  so  $f'$  increasing

i.e.  $(-\infty, -1) \cup (3, \infty)$

concave down  $(-1, 3)$

7. (10 points) Prove carefully that  $x^4 + 3x + 1$  has exactly one root in the interval  $[-2, -1]$ . Make sure to cite any theorems that you use.

$f(x) = x^4 + 3x + 1$  is continuous on  $[-2, -1]$

$$f(-2) = 11$$

$$f(-1) = -1$$

so by Int. Value thm, since  $-1 < 0 < 11$ ,  
there is a root in  $(-2, -1)$

$$\text{Now } f'(x) = 4x^3 + 3$$

If  $f(x)$  had two roots in  $[-2, -1]$ , by Rolle's thm

we would have  $f'(x) = 0$  for some  $x$  in  $[-2, -1]$

$$\text{But } 0 = 4x^3 + 3 \Rightarrow x^3 = -3/4$$

$$\Rightarrow x = \sqrt[3]{-3/4}$$

has one root and it is not in the interval

So only one root!