1. **(15 points)** True or false:

   a. A function which is continuous at $x = a$ must also be differentiable at $x = a$.
   
   b. It is possible for the graph of a function to have 3 vertical asymptotes.
   
   c. The intermediate value theorem applies to $f(x) = 1/x$ on the interval $[-2, 1]$.
   
   d. If $\lim_{x \to 0} f(x) = \infty$ and $\lim_{x \to 0} g(x) = \infty$ then $\lim_{x \to 0} [f(x) - g(x)] = 0$
   
   e. If $p(x)$ is a polynomial then $\lim_{x \to 5} p(x) = p(5)$.

2. **(20 points)**
   
   a. Give the formal definition for $\lim_{x \to a} f(x) = L$.

   b. Use the definition to prove that

   $$\lim_{x \to 4} (3x - 7) = 5.$$
3. (20 points) Evaluate the following limits. If the limit does not exist then write DNE.

a. \( \lim_{x \to -3} \frac{x^2 - 9}{x^2 + 2x - 3} \)

b. \( \lim_{x \to 0} \frac{|x|}{x} \)

c. \( \lim_{x \to -\infty} \frac{\sqrt{x^2 - 9}}{2x - 6} \)

d. \( \lim_{x \to \infty} \frac{2x^2 - 5x + 11}{x^2 - 2} \).
4. (15 points) a. Neatly sketch the graph of a single function $f(x)$ which has the following properties:

- $\lim_{x \to 3^+} f(x) = 2, \quad \lim_{x \to 3^-} f(x) = 0, \quad f(3) = 1.$
- $f(x)$ is continuous from the right at $x = 5$ but not continuous from the left at $x = 5$.
- $\lim_{x \to \infty} f(x) = 4, \quad \lim_{x \to -\infty} f(x) = -1.$

b. Neatly sketch the graph of a single function $g(x)$ which has the following properties:

- $g(x)$ is continuous on $(-\infty, \infty)$
- $g'(6) = 0$
- $g(x)$ is not differentiable at $x = 1$
- $g(x)$ has a vertical tangent line at $x = -5.$
5. (20 points) Let $f(x) = \frac{1}{x}$.
   
a. Use the definition of the derivative to prove that $f'(x) = -\frac{1}{x^2}$.
   
b. Find the equation of the tangent line to $y = \frac{1}{x}$ at the point where $x = 5$.

6. (10 points) The graph of a function $f(x)$ is given below. Use it to sketch the graph of the derivative $f'(x)$ on the same axes.