Name: **SOLUTIONS**

Math 141- Midterm Exam #1 - September 24, 2007

1. (15 points) True or false:
   
   a. A function which is continuous at \( x = a \) must also be differentiable at \( x = a \).
   
   b. It is possible for the graph of a function to have 3 vertical asymptotes.
   
   c. The intermediate value theorem applies to \( f(x) = 1/x \) on the interval \([-2,1]\).
   
   d. If \( \lim_{x \to 0} f(x) = \infty \) and \( \lim_{x \to 0} g(x) = \infty \) then \( \lim_{x \to 0} [f(x) - g(x)] = 0 \)
   
   e. If \( p(x) \) is a polynomial then \( \lim_{x \to 5} p(x) = p(5) \).

2. (20 points)
   
   a. Give the formal definition for \( \lim_{x \to a} f(x) = L \).

   For any \( \varepsilon > 0 \) there is a \( \delta > 0 \) such that:

   if \( 0 < |x - a| < \delta \) then \( |f(x) - L| < \varepsilon \).

   b. Use the definition to prove that

   \( \lim_{x \to 4} (3x - 7) = 5 \).

   Let \( \varepsilon > 0 \) be given. Choose \( \delta = \varepsilon/3 \).

   Suppose \( 0 < |x - 4| < \delta \). Then

   \[
   |f(x) - L| = |3x - 7 - 5| = |3x - 12| = 3|x - 4| < 3 \delta = \varepsilon.
   
   Thus \( |f(x) - L| < \varepsilon \) as required.
3. (20 points) Evaluate the following limits. If the limit does not exist then write DNE.

a. \( \lim_{x \to -3} \frac{x^2 - 9}{x^2 + 2x - 3} \) = \( \lim_{x \to -3} \frac{(x+3)(x-3)}{(x+3)(x-1)} \) = \( \lim_{x \to -3} \frac{x-3}{x-1} \) = \( \frac{-6}{-4} = \frac{3}{2} \)

b. \( \lim_{x \to 0} \frac{|x|}{x} \)

\( \text{DNE} \)  

DNE

c. \( \lim_{x \to -\infty} \frac{\sqrt{x^2 - 3}}{2x - 6} \)  
   For \( x < 0 \), \( x = -\sqrt{x^2} \)
   \( = \lim_{x \to -\infty} \frac{\sqrt{x^2 - 3}}{x} = \lim_{x \to -\infty} \frac{-\sqrt{1 - 9/x^2}}{2 - 6/x} \)
   \( = (-1/2) \)

d. \( \lim_{x \to \infty} \frac{2x^2 - 8x + 11}{x^2 - 2} \)
   \( = 2 \)
4. (15 points) a. Neatly sketch the graph of a single function \( f(x) \) which has the following properties:

- \( \lim_{x \to 3^+} f(x) = 2 \), \( \lim_{x \to 3^-} f(x) = 0 \), \( f(3) = 1 \).
- \( f(x) \) is continuous from the right at \( x = 5 \) but not continuous from the left at \( x = 5 \).
- \( \lim_{x \to -\infty} f(x) = 4 \), \( \lim_{x \to -\infty} f(x) = -1 \).

b. Neatly sketch the graph of a single function \( g(x) \) which has the following properties:

- \( g(x) \) is continuous on \((-\infty, \infty)\)
- \( g'(6) = 0 \)
- \( g(x) \) is not differentiable at \( x = 1 \)
- \( g(x) \) has a vertical tangent line at \( x = -5 \).
5. (20 points) Let \( f(x) = 1/x \).

   a. *Use the definition of the derivative to prove that* \( f'(x) = -1/x^2 \).

   \[
   f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{x - (x+h)}{x(x+h)h} = \frac{-h}{hx} = \lim_{h \to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}
   \]

   b. Find the equation of the tangent line to \( y = 1/x \) at the point where \( x = 5 \).

   \[
   \text{slope} = -\frac{1}{25} = f'(5) \quad \text{point} = (5, \frac{1}{5})
   \]

   \[
   y - \frac{1}{5} = -\frac{1}{25} (x - 5)
   \]

6. (10 points) The graph of a function \( f(x) \) is given below. Use it to sketch the graph of the derivative \( f'(x) \) on the same axes.