Name: SOLUTIONS

Math 141- Midterm Exam #3 - November 12, 2007

1. **(20 points)** Let $f(x) = \frac{x}{x^2+1}$. Find the global maximum and global minimum values of $f(x)$ on the interval $[0, 2]$.

$$f' = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} \qquad \text{C.P. } x = \pm 1$$

x	$f(x)$	
0	0	← Global min value = 0
1	1/2	← Global max value = 1/2
2	2/5	

2. **(5 points)** Complete the following definition. A function $f(x)$ has a *local minimum* at $x = c$ if $\cdots$.

There is $\varepsilon > 0$ so

$f(x) \geq f(c)$ for all x in $(c-\varepsilon, c+\varepsilon)$

3. **(15 points)**
Let $f(x) = \frac{x}{x+2}$. Verify that $f(x)$ satisfies the hypotheses of the Mean Value Theorem on the interval $[1, 4]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

f(x) is continuous on [1,4] and diff'ble on (1,4)
so MVT applies

$$\frac{f(4)-f(1)}{4-1} = \frac{\frac{2}{3}-\frac{1}{3}}{3} = 1/9$$

$$f'(x) = \frac{x+2-x}{(x+2)^2} = \frac{2}{(x+2)^2}$$

set $\frac{2}{(x+2)^2} = \frac{1}{9}$

$$18 = (x+2)^2$$

$$x+2 = \pm 3\sqrt{3}$$

$$x = -2 \pm 3\sqrt{3}$$

$$\boxed{c = -2+3\sqrt{3}}$$

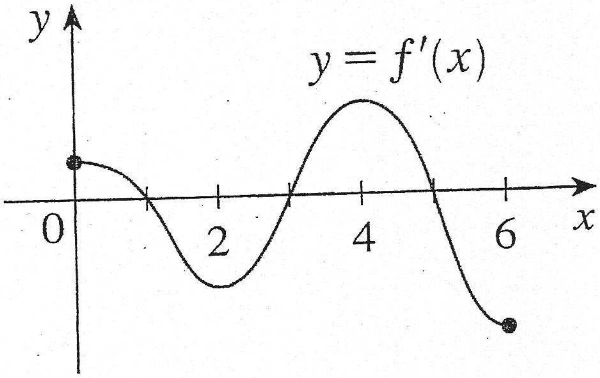
$-2-3\sqrt{3}$ not in $(1,4)$

4. **(15 points)** The graph of the *derivative* f' of a function f is shown.

a. On what intervals is f increasing or decreasing?

b. At what values of x does f have a local maximum or minimum?

c. At what values of x does the graph of $f(x)$ have inflection points?



a. Increasing $(0,1) \cup (3,5)$

Decreasing $(1,3) \cup (5,6)$

b. Local max at $x = 1, 5$

Local min at $x = 3$

c. Inflection points at $x = 2, 4$

5. (**20 points**) Let

$$f(x) = (x^2 - 1)^{2/3}.$$

Then:

$$f'(x) = \frac{4}{3}\frac{x}{(x^2-1)^{1/3}}, \qquad f''(x) = \frac{4}{9}\frac{(x^2-3)}{(x^2-1)^{4/3}}.$$

a. Find all x and y intercepts and any asymptotes.

b. Find the intervals where $f(x)$ is increasing or decreasing and any local maximums or local minimums.

c. Find the intervals where $f(x)$ is concave up or concave down, and determine any inflection points.

d. Neatly sketch the graph of $y = f(x)$, Label the x and y coordinates of any intercepts, local extrema and inflection points.

a. $(0,1)$ $(1,0)$ $(-1,0)$

~~V.A. $x=1$, $x=-1$~~ No H.A, No V.A.

b.

$$\begin{array}{ccccccccc} & - & & + & & - & & + & \quad f'(x) \\ \hline & & -1 & & 0 & & 1 & & \end{array}$$

Decreasing $(-\infty,-1) \cup (0,1)$

Increasing $(-1,0) \cup (1,\infty)$

Local min $(-1,0)$, $(1,0)$

Local max $(0,1)$

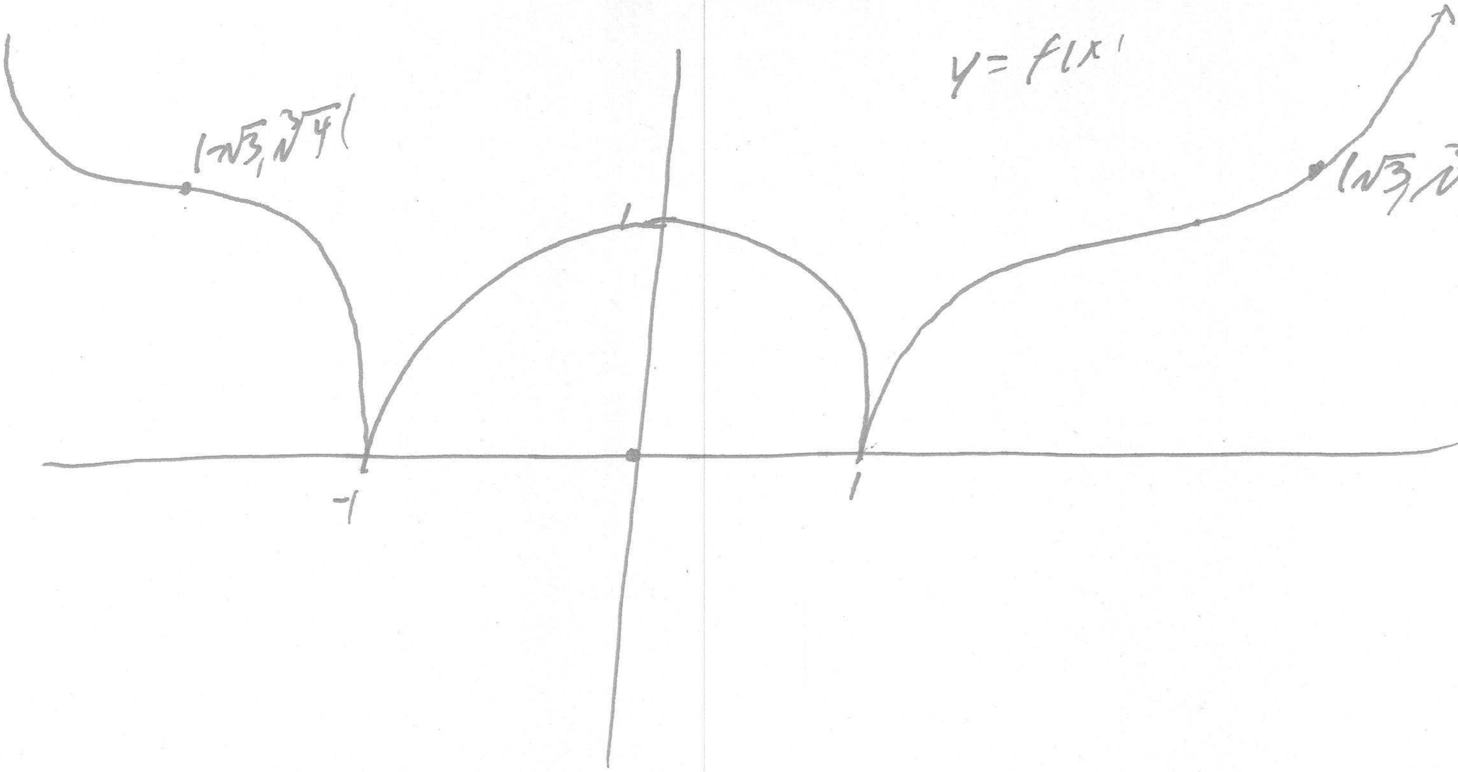
c.

$$\begin{array}{ccccccccccc} & + & & - & & - & & - & & + & \quad f''(x) \\ \hline & & -\sqrt{3} & & -1 & & 1 & & \sqrt{3} & & \end{array}$$

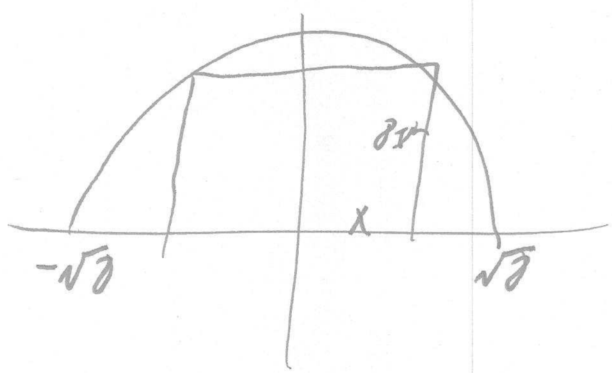
concave up $(-\infty,-\sqrt{3}) \cup (\sqrt{3},\infty)$

concave down $(-\sqrt{3},-1) \cup (-1,1) \cup (1,\sqrt{3})$

I.P. $(-\sqrt{3}, \sqrt[3]{4})$, $(\sqrt{3}, \sqrt[3]{4})$



6. **(15 points)** Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 8 - x^2$.



Maximize $A(x) = 2x(8-x^2) \quad 0 \le x \le \sqrt{8}$

$= 16x - 2x^3$

$A'(x) = 16 - 6x^2$

$x = \pm \sqrt{\frac{8}{3}}$

$\sqrt{8/3}$

$x = \quad y = 8 - \frac{8}{3} = 16/3$

Dimensions $\boxed{2\sqrt{8/3} \times 16/3}$

7. **(10 points)** Evaluate the following limits:

$$a.\ \lim_{x\to 0^+} \frac{\cos x}{x}, \qquad b.\ \lim_{x\to\infty} (e^x + x)^{1/x}$$

a. ∞ [LHR does not apply]

b. $y = (e^x + x)^{1/x}$

$\ln y = \frac{1}{x} \ln(e^x + x)$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(e^x+x)}{x} \overset{LHR}{=} \lim_{x\to\infty} \frac{\frac{e^x+1}{e^x+x}}{1}$$

$$\overset{LHR}{=} \lim_{x\to\infty} \frac{e^x}{e^x+1}$$

$$\overset{LHR}{=} \lim_{x\to\infty} \frac{e^x}{e^x} = 1$$

Thus $\lim_{x\to\infty} y = e^1 = e$