NAME:

Math 141A- Final Exam - December 8, 2014

Instructions: The exam is worth 150 points. You should not use any aids, electronic or paper, other than a writing utensil.

1. **(15 points)** Evaluate the following integrals:

   a. $\int_0^\pi \sin(3t)dt$.

   b. $\int \frac{4x+2}{x^2+x-1}dx$.

   c. $\int_0^2 y^2\sqrt{1+y^3}dy$

   d. $\int x^3 + 2x^2 + \frac{1}{x}dx$.

   e. $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

2. **(15 points)** a. Estimate the area under the curve $y = 1 + x^2$ for $0 \leq x \leq 2$ using 4 intervals and right hand endpoints (i.e. compute R_4.) Is your answer an overestimate or underestimate? Explain.

b. Now find the exact area by computing the following definite integral **using the definition as a limit of Riemann sums!**:

$$\int_0^2 1 + x^2 dx.$$

You may find one of the following formulas useful:
$\sum_{i=1}^n i = \frac{n(n+1)}{2}$, $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$.

3. **(10 points)** State both parts of the Fundamental Theorem of Calculus.

4. **(10 points)** Use the Fundamental Theorem of Calculus to find the derivative $g'(x)$:

a. $g(x) = \int_3^x e^{t^2} dt$.

b. $g(x) = \int_0^{x^2} \cos(t) dt$.

5. **(10 points)** Consider the following limit:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{1+(i/n)^2} \cdot \frac{1}{n}.$$

Recognize this limit as a definite integral and then evaluate it by evaluating the integral.

6. **(10 points)** Let $f(t) = t\sqrt{4-t^2}$. Find the absolute maximum and absolute minimum of $f(t)$ on the interval $[-1, 2]$.

7. (**5 points**) Using the Fundamental Theorem of Calculus we conclude:

$$\int_{-1}^{2} 3x^{-4}dx = -x^{-3} \mid_{-1}^{2} = -\frac{1}{x^3} \mid_{-1}^{2} = -(1/8+1) = -9/8.$$

Why is this mistaken?

8. **(5 points)** If $f(x)$ represents the slope of a trail at a distance x miles from the start, what does $\int_2^6 f(x)dx$ represent?

9. **(10 points)** The formula for the derivative of a function $f(x)$ at the value $x = a$ is given by:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if it exists. Explain in words, and using diagrams, where this formula comes from.

10. **(10 points)** Let $f(x) = x^3 + 2x + 1$.

a. Find the equation of the tangent line at $x = 2$.

b. Prove $f(x)$ has one real root.

11. **(10 points)** Complete the definitions:

a. $\lim_{x \to a} f(x) = L$ if ...

b. A function $f(x)$ is *continuous* at $x = a$ if ...

12. **(10 points)** Find $\frac{dy}{dx}$.

a. $y = xe^x$

b. $y = 6x^2 + 3x + 5$

c. $y = (1 + \cos(x))^{10}$

d. $y = \frac{1+x}{1-\ln(x)}$

e. $xy^2 + \cos(y) = 2x$

13. **(15 points)**

a. Find the equation of the line passing through $(-2, 1)$ and parallel to the line $2x + 3y = 2$.

b. Solve the inequality $x^2 - x - 6 \geq 0$.

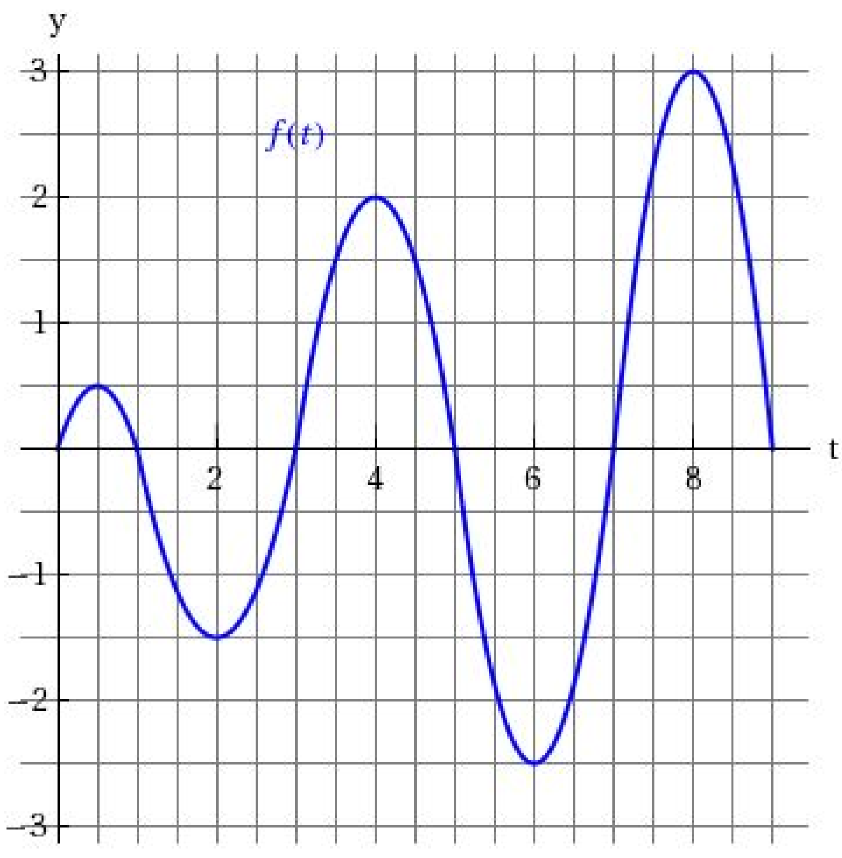
c. Find $\log_3(27)$.

d. Write $\frac{\cos x}{1+x} + \frac{x}{2+x}$ as a single fraction.

e. Neatly sketch and label the graph of $y = 2\sin x$ and $y = 3 + \ln x$ on the same axes.

14. **(5 points)** Let $f(x) = 2x^2 + 3x + 1$. Find an x value c such that c satisfies the conclusion of the Mean Value Theorem for $f(x)$ on the interval $[1, 4]$.

15. **(10 points)** Below is the graph of $y = f(t)$ for $0 \leq t \leq 27$. (Please note the scale on the axis, each unit is 1.5)



a. For what intervals is $f(t)$ increasing or decreasing? Concave up or concave down?

b. Estimate $f'(15)$.

c. Estimate $\int_0^6 f(t)dt$.

d. Now define $g(x) = \int_0^x f(t)dt$. For what intervals is $g(x)$ increasing or decreasing?

e. What is $g'(12)$?

f. At what values does $g(x)$ obtain local maximums? Global maximum?