Math 141A- Fall 2014- Midterm 1

Problem #	Max Score	Score
1	15	
2	20	
3	20	
4	10	
5	20	
6	15	

TOTAL

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Math 141A- Midterm Exam #1 - September 25, 2014

1. **(15 points)** True or false:

____ a. A function which is differentiable at $x = a$ must also be continuous at $x = a$.

____ b. The composition of two continuous functions is also continuous.

____ c. Suppose $f(x)$ is continuous on $[0, 2]$ and $f(0) = 2$, $f(2) = 5$. Then the intermediate value theorem implies $f(x)$ does not have a root in $(0, 2)$.

____ d. Suppose $y = a$ is a horizontal asymptote for $f(x)$. Then the graph of $y = f(x)$ does not cross the line $y = a$.

____ e. $f(x) = \frac{\sin x}{x}$ has a removable discontinuity at $x = 0$.

2. **(20 points)**
   a. Give the $\epsilon - \delta$ definition for $\lim_{x \to a} f(x) = L$.

   b. Use the definition to prove that

   $$\lim_{x \to 3} (2x + 8) = 14.$$

3. (**20 points**) Evaluate the following limits. If the limit does not exist then write DNE.

a. $\lim_{x \to -\infty} \frac{\sqrt{x^2+5}}{2x-3}$

b. $\lim_{x \to 3^+} \frac{5-3x}{(x-3)(x-5)}$

c. $\lim_{x \to 6} \sin x$.

d. Suppose $\lim_{x \to 2} f(x) = 3$ and $\lim_{x \to 2} g(x) = -1$. Evaluate $\lim_{x \to 2} \frac{f(x)+3(g(x)^2)}{\sqrt{f(x)}}$.

4. **(10 points)** Neatly sketch the graph of a single function $f(x)$ which has the following properties:

- $\lim_{x \to 3^+} f(x) = 2$, $\lim_{x \to 3^-} f(x) = \infty$, $f(3) = 1$.

- $f(x)$ is continuous from the right at $x = 5$ but not continuous from the left at $x = 5$.

- $\lim_{x \to \infty} f(x) = 4$, $\lim_{x \to -\infty} f(x) = -1$.

- $f'(6) = 0$.

5. **(20 points)** A ball is tossed and has height in feet given at time t seconds by $y(t) = -t^2 + 6t$.
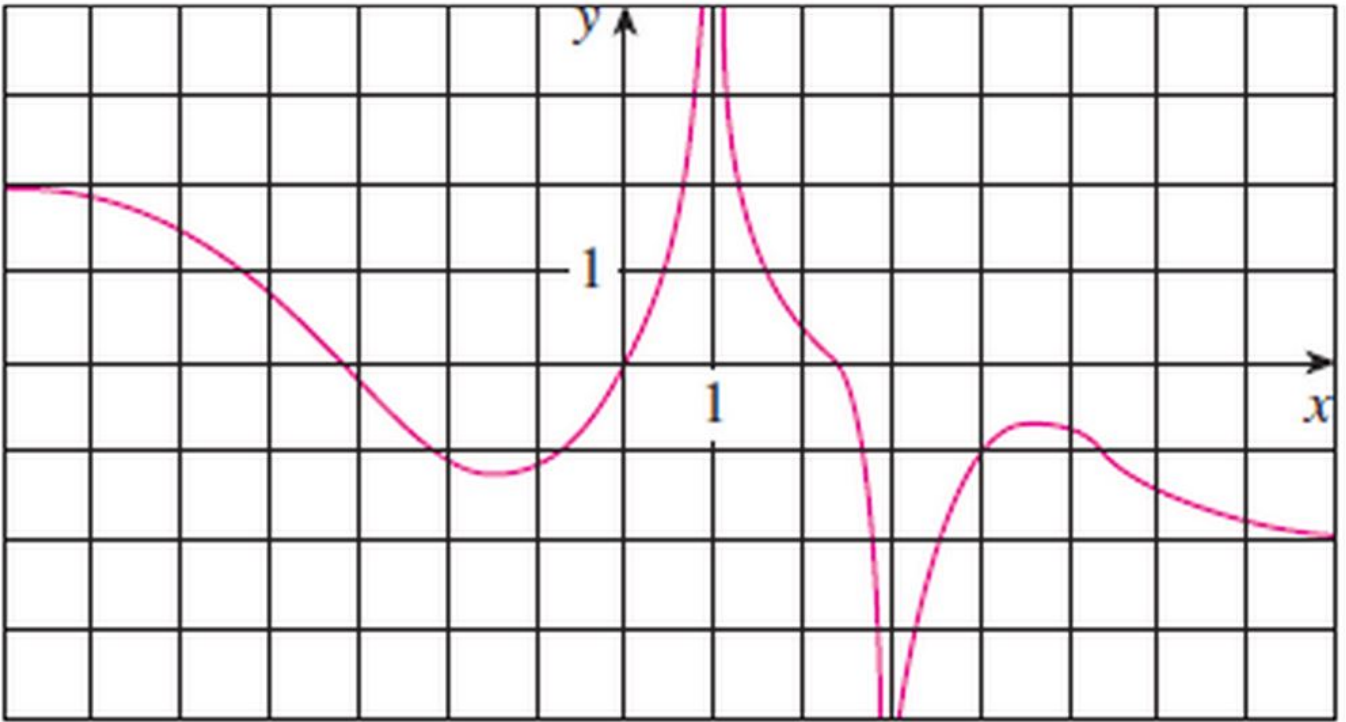
   a. *Use the definition* of the derivative to prove that $y'(t) = -2t + 6$.

   b. Find the equation of the tangent line to $y = h(t)$ at the point where $t = 2$.

   c. What is the ball's average velocity from time $t = 1$ to time $t = 3$?

   d. How fast is the ball moving when it hits the ground?

   e. What is the ball's acceleration?

6. **(15 points)** Above is the graph of a function $y = f(x)$.
   a. Find the vertical and horizontal asympotes.

   b. Evaluate $\lim_{x \to \infty} f(x)$.

   c. Estimate $f'(0)$.

   d. Estimate $\lim_{x \to \infty} f'(x)$.

   e. On the same axes above carefully sketch a graph of $y = f'(x)$

Name: