Math 141A- Fall 2014- Midterm 1

<table>
<thead>
<tr>
<th>Problem #</th>
<th>Max Score</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td></td>
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<tr>
<td>2</td>
<td>20</td>
<td></td>
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<td>3</td>
<td>20</td>
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<td>4</td>
<td>10</td>
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<td>5</td>
<td>20</td>
<td></td>
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<td>6</td>
<td>15</td>
<td></td>
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</tbody>
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1. (15 points) True or false:

_____ a. A function which is differentiable at \( x = a \) must also be continuous at \( x = a \).

_____ b. The composition of two continuous functions is also continuous.

_____ c. Suppose \( f(x) \) is continuous on \([0, 2]\) and \( f(0) = 2, f(2) = 5 \). Then the intermediate value theorem implies \( f(x) \) does not have a root in \((0, 2)\).

_____ d. Suppose \( y = a \) is a horizontal asymptote for \( f(x) \). Then the graph of \( y = f(x) \) does not cross the line \( y = a \).

_____ e. \( f(x) = \frac{\sin x}{x} \) has a removable discontinuity at \( x = 0 \).

2. (20 points)

a. Give the \( \epsilon - \delta \) definition for \( \lim_{x \to a} f(x) = L \).

b. Use the definition to prove that

\[
\lim_{x \to 3} (2x + 8) = 14.
\]
3. (20 points) Evaluate the following limits. If the limit does not exist then write DNE.

a. \( \lim_{x \to -\infty} \frac{\sqrt{x^2 + 5}}{2x - 3} \)

b. \( \lim_{x \to 3^+} \frac{5 - 3x}{(x - 3)(x - 5)} \)

c. \( \lim_{x \to 6} \sin x \).

d. Suppose \( \lim_{x \to 2} f(x) = 3 \) and \( \lim_{x \to 2} g(x) = -1 \). Evaluate \( \lim_{x \to 2} \frac{f(x) + 3(g(x))^2}{\sqrt{f(x)}} \).
4. **(10 points)** Neatly sketch the graph of a single function $f(x)$ which has the following properties:

- $\lim_{x \to 3^+} f(x) = 2$, $\lim_{x \to 3^-} f(x) = \infty$, $f(3) = 1$.
- $f(x)$ is continuous from the right at $x = 5$ but not continuous from the left at $x = 5$.
- $\lim_{x \to \infty} f(x) = 4$, $\lim_{x \to -\infty} f(x) = -1$.
- $f'(6) = 0$. 
5. **(20 points)** A ball is tossed and has height in feet given at time \( t \) seconds by \( y(t) = -t^2 + 6t \).

a. *Use the definition* of the derivative to prove that \( y'(t) = -2t + 6 \).

b. Find the equation of the tangent line to \( y = h(t) \) at the point where \( t = 2 \).

c. What is the ball’s average velocity from time \( t = 1 \) to time \( t = 3 \)?

d. How fast is the ball moving when it hits the ground?

e. What is the ball’s acceleration?
6. (15 points) Above is the graph of a function $y = f(x)$.
   a. Find the vertical and horizontal asymptotes.

   b. Evaluate $\lim_{x \to \infty} f(x)$.

   c. Estimate $f'(0)$.

   d. Estimate $\lim_{x \to \infty} f'(x)$.

   e. On the same axes above carefully sketch a graph of $y = f'(x)$.
Name: