

# Lecture 10

Review How can  $f(x)$  fail to be differentiable at  $x=a$ ?

Higher derivatives

$f''(x) = (f'(x))'$  measure rate of change of rate of change...

Ex Acceleration

Ex The deficit is increasing, but not as quickly as before

Shape of curves

Notation  $y = f(x)$   $\frac{d^2y}{dx^2} = y'' = f''(x)$

## Chapter 3 - Differentiation Rules

1.  $\frac{d}{dx}(c) = 0$

pf  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{c - c}{h} = \lim_{h \rightarrow 0} \frac{0}{h} = 0$

Ex  $f(x) = x^2$   $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{x \rightarrow a} \frac{x^2 - a^2}{x - a} = 2a$

Power Rule Let  $n$  be a positive integer.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

pf1  $f(x) = x^n$   $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}$   
 $= \lim_{x \rightarrow a} x^{n-1} + x^{n-2}a + \dots + a^{n-1} = na^{n-1}$

pf2  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h} = \frac{x^n + nx^{n-1}h + h(-) - x^n}{h}$   
 $= nx^{n-1}$  binomial Thm

$$\text{Ex } x(t) = t^5 \quad \dot{x}(t) = 5t^4$$

Generalization  $\frac{d}{dx}(x^n) = n x^{n-1}$  for any real  $\neq n$ .

$$\text{Ex } f(x) = \sqrt{x} \quad f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$$
$$= x^{1/2}$$

$$\text{Ex } f(x) = x\sqrt{x} \quad \text{Find tangent line at } x=9$$

$$\Delta f(x) = x^{3/2} \quad f'(x) = \frac{3}{2} x^{1/2}$$

$$f(9) = 27 \quad f'(9) = 9/2$$

$$\boxed{y - 27 = \frac{9}{2}(x - 9)}$$

Constant Multiple Rule

$$\frac{d}{dx}(c f(x)) = c \frac{d}{dx} f(x)$$

Proof . . .

Sum & <sup>Diff</sup> Product Rule

$$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$$

$$\text{i.e. } (f \pm g)' = f' \pm g'$$

$$\text{Ex } y = 7x^2 - 3\sqrt{x} \quad y' = 14x - \frac{3}{2}x^{-1/2}$$

Ex Let  $y = x^3 - 6x^2 + 4$ . Where on curve is tangent line horizontal?

$$y' = 4x^3 - 12x$$

$$\text{set} = 0 \quad 4x(x^2 - 3) = 0 \quad x = 0, \pm\sqrt{3}$$

$(0, 4)$
$(\sqrt{3}, -5)$
$(-\sqrt{3}, 5)$

Ex  $s(t) = 2t^3 - 4t^2 + t + 1$ . Find acceleration.  
position of particle

$$v(t) = s'(t) = 6t^2 + 8t + 1$$

$$a(t) = v'(t) = s''(t) = 12t + 8$$

### Exponential Functions

• Review graphs of  $y = b^x$ ,  $y = b^{-x}$ , properties of functions

### Problem

$f(x) = b^x$ , what is  $f'(x)$ ?

$$f'(x) = \lim_{h \rightarrow 0} \frac{b^{x+h} - b^x}{h} = \lim_{h \rightarrow 0} \frac{b^x (b^h - 1)}{h}$$

$$= b^x \cdot \lim_{h \rightarrow 0} \left( \frac{b^h - 1}{h} \right)$$

\* Derivative of  $b^x$  is a constant times  $b^x$ . What is this constant?  $f'(0)$ !!

Def  $e$  is the real number so that  $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$

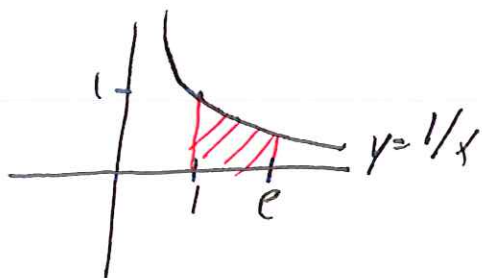
i.e.  $y = b^x$  has  $y'(0) = 1$  for  $b = e$ .

Corollary  $y = e^x$  then  $y'(x) = e^x$ .

called natural exponential function

Rmk 1.  $e \approx 2.718281\dots$  is irrational

2. Later:



 area = 1