Lecture 11 Inverse Functions and Logarithms

Inverse Functions For some functions $y = f(x)$ the function can be "undone"; i.e. y value determines x-value.

Ex 1. $f(x) = x + 3$

2. $f(x) = x^3$

3. $f(\text{student}) = \text{person } \#$

For others there are multiple x values mapping to same y:

Ex 1. $f(x) = x^2$ $\quad 3 \xrightarrow{f} 9$, $\; -3 \to 9$

2. $f(x) = \sin x$ $\quad 0 \to 0$, $\; \pi \to 0$, $\; 2\pi \to 0$ etc.

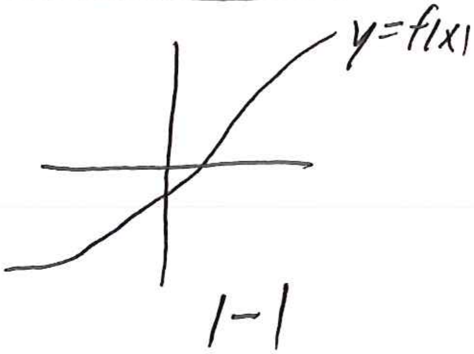
3. $f(\text{person}) = \text{age}$

Def A function f is called one-to-one if it never takes same value twice; that is

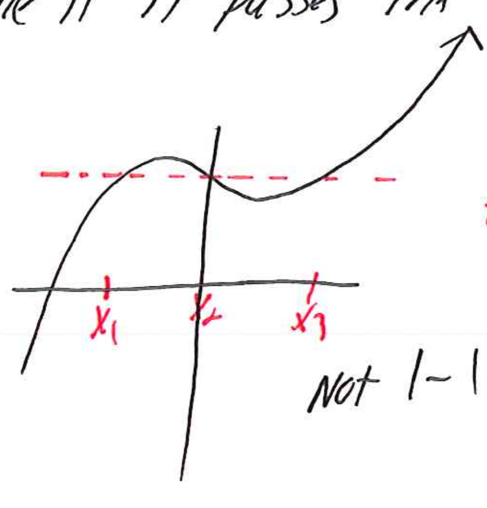
$$\text{If } f(x_1) = f(x_2) \text{ then } x_1 = x_2.$$

Ex $f(x) = x^3$ is one-to-one

Easy Fact $f(x)$ is one to one if it passes the horizontal line test



1-1



Not 1-1

$f(x_1) = f(x_2) = f(x_3)$

fails H.L.T.

Def Suppose $f(x)$ is one-to-one with domain A, range B.

The *inverse function* $f^{-1}(x)$ has domain B, range A, and is defined by $\quad f^{-1}(y) = x \iff f(x) = y$.

* warning: $f^{-1}(x)$ does not mean $\frac{1}{f(x)}$.

Ex 1. $f(x) = x^3 \qquad f^{-1}(x) = x^{1/3}$

2. $f(x) = 6x+3 \qquad f^{-1}(x) = \frac{x-3}{6}$

To solve for f^{-1}

1. Write $y = f(x)$
2. Solve for x
3. swap x & y

Ex $\quad y = x^3$

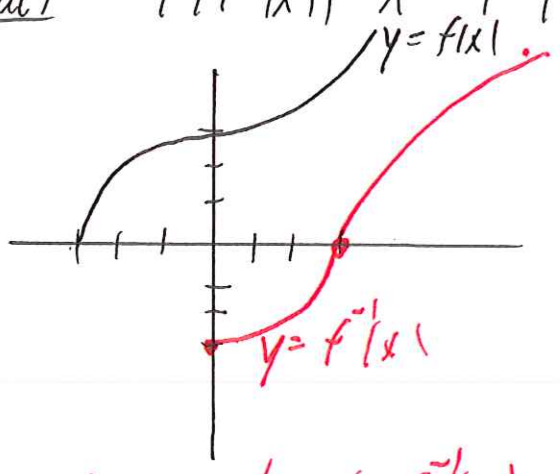
$x = y^{1/3}$

$f^{-1}(x) = x^{1/3}$

Ex $\quad y = 6x+3$

$x = \frac{y-3}{6} \qquad f^{-1}(x) = \frac{x-3}{6}$

Fact $\quad f(f^{-1}(x)) = x = f^{-1}(f(x))$

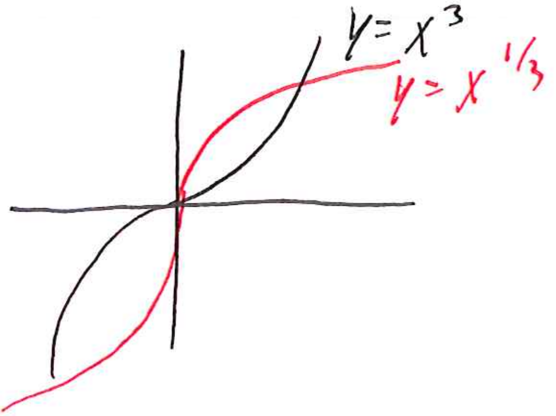


$f(0) = 3 \qquad f^{-1}(3) = 0$

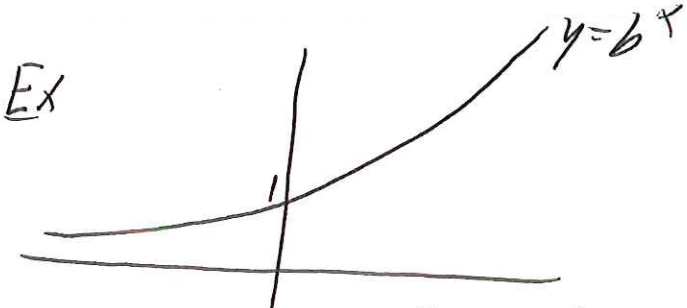
$f(-3) = 0 \qquad f^{-1}(0) = -3$

$f(4) = 7 \qquad f^{-1}(7) = 4$

* graph of $f^{-1}(x)$ is reflection of graph of f across $y = x$.



Ex



Domain $(-\infty, \infty)$
Range $(0, \infty)$

Logarithms

Def Suppose $b \neq 0, 1$. Then $f(x) = b^x$ is 1-1 on $(-\infty, \infty)$.

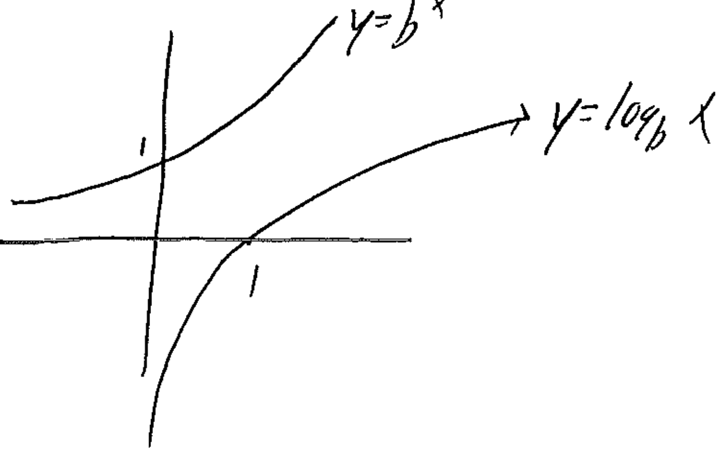
The logarithm base b is the inverse, denoted $\log_b x$.

$$* \quad \log_b x = y \iff b^y = x.$$

$$* \quad \log_b (b^x) = x \qquad x \in (-\infty, \infty)$$

$$b^{\log_b x} = x \qquad x \in (0, \infty)$$

Domain $\log_b x = (0, \infty)$



Ex $\log_2 8 = 3$

$3^{\log_3 6} = 6$

$\log_b(1) = 0$ since $b^0 = 1$

<u>Laws of Logarithms</u>

<u>Corresponding exponentials</u>

$\log_b(xy) = \log_b(x) + \log_b y$ $\qquad$ $b^x b^y = b^{x+y}$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$ $\qquad$ $\frac{b^x}{b^y} = b^{x-y}$

$\log_b(x^r) = r\log_b x$ $\qquad$ $(b^x)^y = b^{xy}$

<u>Ex</u> $\log_2 80 - \log_2 5 = \log_2 16 = \log_2 2^4 = 4$

<u>Natural Logarithm</u>

Instead of $\log_e$ we write $\ln$.

$\ln e^x = x$

$e^{\ln x} = x$