Lecture 12

Review inverse functions, def of $\log_b x$.

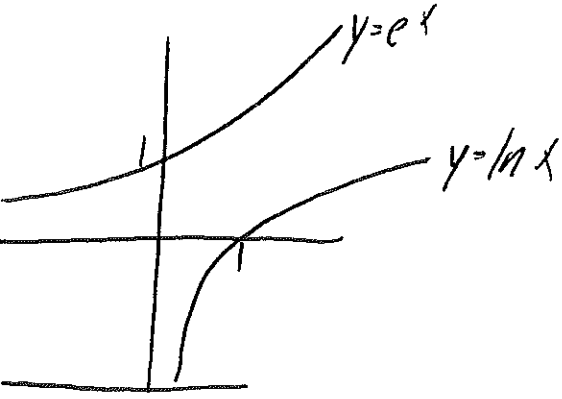
Examples $\log_3 9 = 2$, $3^{\log_3 5} = 5$, $\log_b(1) = 0$.

Rmk Laws for exponentials give log laws:

1. $b^x b^y = b^{x+y} \longrightarrow \log_b(xy) = \log_b x + \log_b y$
2. $\frac{b^x}{b^y} = b^{x-y} \longrightarrow \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
3. $(b^x)^r = b^{xr} \longrightarrow \log_b(x^r) = r \log_b x$

Ex $\log_2 80 - \log_2 5 = \log_2 16 = 4$

Natural logs Write $\ln x$ instead of $\log_e x$.



$\ln e = 1$

* Applying ln brings exponents down.

Ex $\ln 10 + 2\ln 5$, write as a single log

Ex Solve $3^{5+2x} = 10$

$\ln(3^{5+2x}) = \ln 10$

$(5+2x)\ln 3$

$5+2x = \frac{\ln 10}{\ln 3}$ $\quad x = \left(\frac{\ln 10}{\ln 3} - 5\right) \cdot \frac{1}{2}$

Ex Find domain of $y = \ln(x^2 - 9)$

Ex $\ln(x^2 - 1) = 3$ solve for x.

Ex $1 < e^{3x-1} < 2$ solve for x.

Change of Base

Let $y = \log_b x$. Then $b^y = x$ so $\ln(b^y) = \ln x$

$y \ln b$

so $y = \frac{\ln x}{\ln b}$

Change of base formula → $\boxed{\log_b x = \frac{\ln x}{\ln b}}$

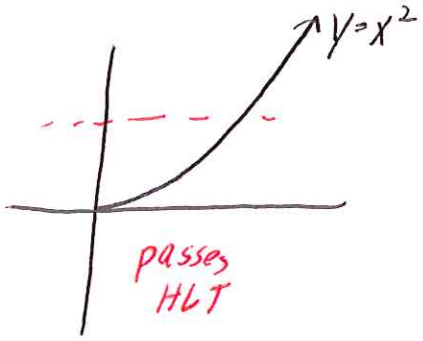
all $\log_b$ functions are just multiples of $\ln x$.

Ex Find domain, sketch graph of $y = |\ln(x-3) + 4|$

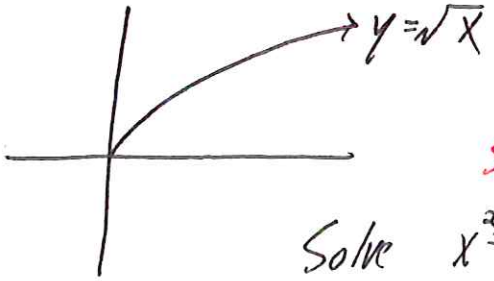
Ex $y = e^{2x-1}$. Find inverse function

Inverse Trig Functions

Rmk $f(x) = x^2$ is not 1-1. However if we restrict domain to $[0,\infty)$ then it is:



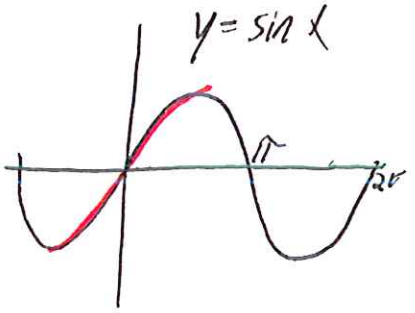
Define $f(x) = \sqrt{x}$ as unique # in $[0,\infty)$ so squared gives x



arbitrary choice!

So $\sqrt{9}=3$

Solve $x^2=9 \rightarrow x=\pm\sqrt{9}$

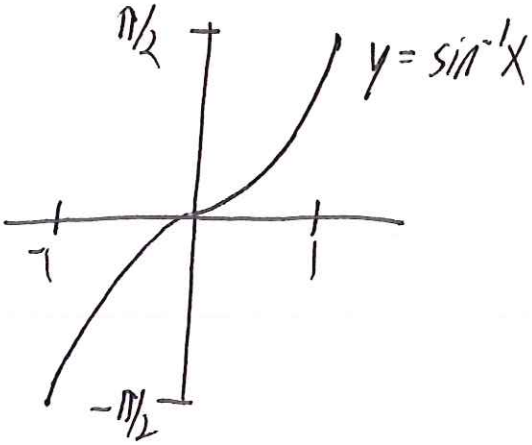


* $\sin x$ is 1-1 for $-\pi/2 \le x \le \pi/2$

Define inverse sine function or arcsine by:

$$\sin^{-1} x = y \iff \sin y = x \quad \text{and} \quad -\pi/2 \le y \le \pi/2$$

Compare $\sqrt{x}=y \iff x=y^2$ and $0 \le y$



Domain $\sin^{-1} x = [-1,1]$

Range $= [-\pi/2, \pi/2]$

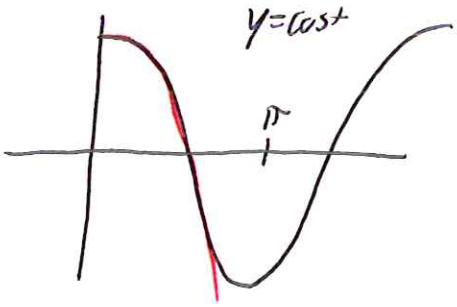
Ex $\sin^{-1}(1/2)$

$\sin^{-1}(1/2) = \pi/6$

Ex $\sin^{-1}(\sin 7\pi/6) = \sin^{-1}(1/2) = \pi/6$

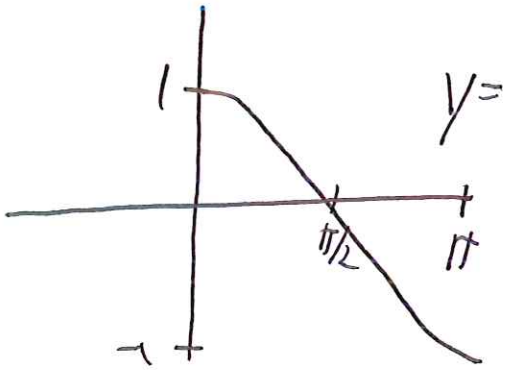
* $\sin^{-1}(\sin x) = x$ if $-\pi/2 \le x \le \pi/2$

Compare $\sqrt{x^2} = x$ if $0 \le x$



Choose $[0, \pi]$

$\cos^{-1} x = y \leftrightarrow \cos y = x$ and $0 \le y \le \pi$



$y = \cos^{-1} x$

Finally $\tan x$ is 1-1 if $-\pi/2 < x < \pi/2$

$\tan^{-1} x = y \leftrightarrow x = \tan y$ and $-\pi/2 < y < \pi/2$