Lecture 14

Review: Product Rule, Quotient Rule.

Ex $y = \frac{x^2-3x+1}{x^3+6x^2+1}$ find y'.

Ex $y = \frac{3x^2+2\sqrt{x}}{x}$ ← easier to simplify 1st.

Ex Find equations of tangent lines to $y = \frac{x-1}{x+1}$ parallel to $x-2y=2$.

TRIG FUNCTIONS

Let's try to compute $\frac{d}{dx}(\sin x)$

$$= \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h} = \lim_{h\to 0} \frac{\sin x\cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \sin x\left(\frac{\cos h-1}{h}\right) + \lim_{h\to 0} \frac{\cos x \sin h}{h}$$

$$= \sin x \cdot \underbrace{\lim_{h\to 0} \frac{\cos h-1}{h}}_{??} + \cos x \underbrace{\lim_{h\to 0} \frac{\sin h}{h}}_{??}$$

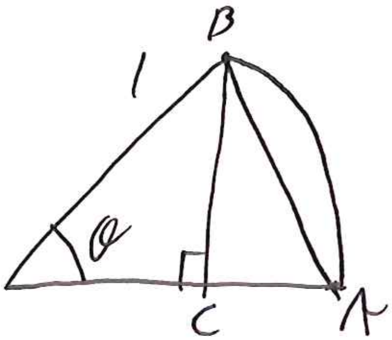
Thm

$$\lim_{h\to 0} \frac{\sin h}{h} = 1$$

Pf Use Sqz Thm

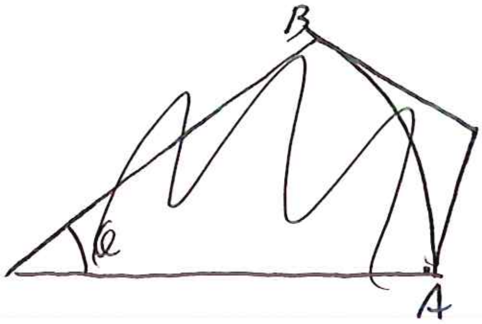
Diagrams For Sqz Thm

Suppose $0 < \theta < \pi/2$, consider unit circle

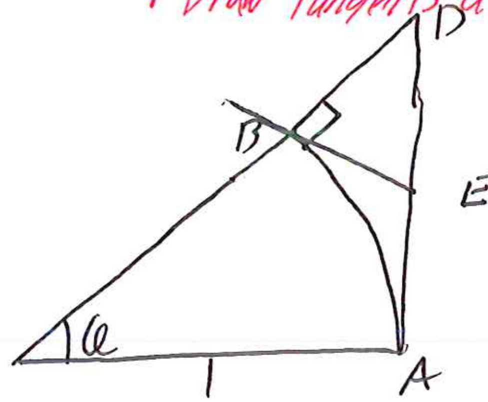


$\sin\theta = |BC| < |BA| < |\widehat{AB}| = \theta$

so $\boxed{\frac{\sin\theta}{\theta} < 1}$



. Draw tangents at B & A



$\theta = \text{arc } AB < |AE| + |BE| < |AE| + |ED| = |AD| = \tan\theta$

so $\theta < \tan\theta = \frac{\sin\theta}{\cos\theta}$ so $\cos\theta < \frac{\sin\theta}{\theta}$

$* \cos\theta < \frac{\sin\theta}{\theta} < 1$

So by squeeze thm $\lim_{\theta \to 0^+} \frac{\sin\theta}{\theta} = 1$

but $\frac{\sin(-\theta)}{-\theta} = \frac{\sin\theta}{\theta}$ so $\boxed{\lim_{\theta \to 0^+} \frac{\sin\theta}{\theta} = 1}$

Cor $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$

Pf $= \lim_{\theta \to 0} \frac{\cos^2\theta - 1}{\theta(\cos\theta + 1)} = \lim_{\theta \to 0} \frac{\sin^2\theta}{\theta(\cos\theta+1)}$

$= \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \lim_{\theta \to 0} \frac{\sin\theta}{\cos\theta + 1}$

$= 1 \cdot 0 = 0.$

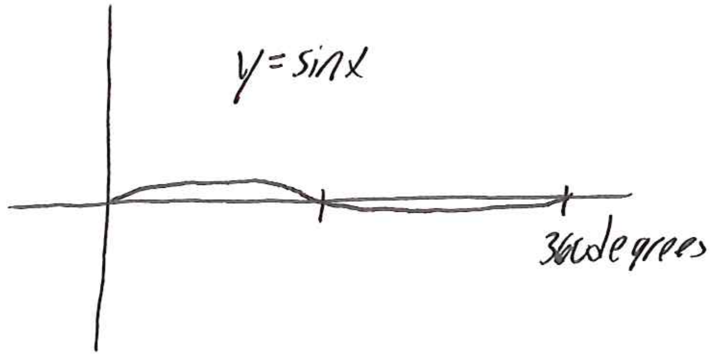
Conclude $$\boxed{\frac{d}{dx}(\sin x) = \cos x}$$

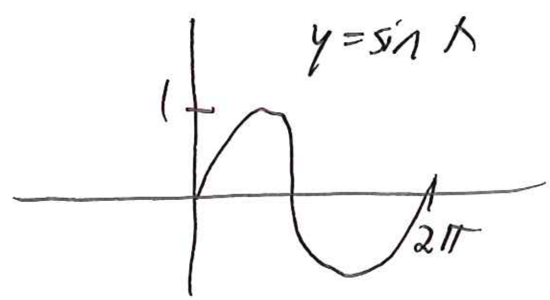
• check graphs

HW Use same two limits plus formula for $\cos(x+h)$ to get

$$\boxed{\frac{d}{dx}(\cos x) = -\sin x}$$

* Warning: limit computations assumed θ in radians





Rest from Quotient Rule

$\tan x = \frac{\sin x}{\cos x}$ $\quad \frac{d}{dx}(\tan x) = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x}$

$$\boxed{\frac{d}{dx}\tan x = \sec^2 x}$$

$$\boxed{\frac{d}{dx}\cot x = -\csc^2 x}$$

$\sec x = \frac{1}{\cos x}$ $\quad \frac{d}{dx}(\sec x) = \frac{-1(\sin x)}{\cos^2 x} = \sec x \tan x$

$$\boxed{\begin{array}{l} \frac{d}{dx}\sec x = \sec x \tan x \\ \frac{d}{dx}\csc x = -\csc x \cot x \end{array}}$$

Problems

1. Find tang line to $y = 3x + 6\cos x$ at $(\pi/3, \pi + 3)$
2. $f(x) = e^x \cos x$. Find $f''(x)$
3. $f(x) = \sin x, \cos x$ Find $f''(x)$
4. Find 17^{th} derivative of $\sin x$
5. (EX #3) Object at end of spring stretched 4cm & released. Position at time t is $f(t) = 4\cos t$. Find $v(t), a(t)$ and analyze.