Lecture 15

Recall $\frac{d}{dx}(\sin x) = \cos x$ via application of squeeze thm to prove $\lim_{h\to 0} \frac{\sin h}{h} = 1$, $\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$.

Similarly $\frac{d}{dx}(\cos x) = -\sin x$. Now use quotient rule:

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

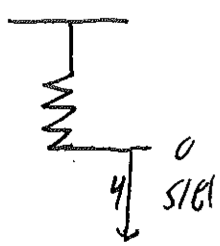
$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

Problems

1. Find tan line to $y = 3x + 6\cos x$ at $(\pi/3, \pi + 3)$

2. $f(x) = x^2 \sin x$. Find $f'(x)$ & $f''(x)$.

3. Object at end of a spring stretched 4 cm beyond rest & released



position is $s = f(t) = 4\cos t$

- Find $v(t)$ & $a(t)$
- Analyze when $\vec{v}(t), \vec{a}(t)$ are zero or max.

4. $y = e^x \cos x$. Find tangent line at $x = 0$.

5. $f(t) = te^t \cot t$. Find $f'(t)$.

Some more limits

Recall $\displaystyle\lim_{h\to 0} \frac{\sin h}{h} = 0.$

1. Find $\displaystyle\lim_{x\to 0} \frac{\sin(5x)}{3x}$

2. $\displaystyle\lim_{x\to 0} \frac{\sin x}{\sin(\pi x)}$. mult by $\dfrac{\pi x}{x} \dfrac{1}{\pi}$

3. $\displaystyle\lim_{t\to 0} \frac{\tan 6t}{\sin 2t} = \lim_{t\to 0} \frac{\sin 6t}{\cos(6t)\sin 2t} \cdot \frac{2t\cdot 3}{6t} = \lim \frac{\sin 6t}{6t}\frac{2t}{\sin 2t}\frac{1}{\cos 6t}3$

$= 3$

4. $\displaystyle\lim_{x\to \pi/4} \frac{1-\tan x}{\sin x-\cos x} = \lim_{x\to \pi/4} \frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x} = \lim_{x\to \pi/4} \frac{\cos x-\sin x}{(\sin x-\cos x)\cos x}$

$\displaystyle = \lim_{x\to \pi/4} \frac{-1}{\cos x} = -\sqrt{2}$

5. Find constants A & B so

$$y = A\sin x + B\cos x \quad \text{satisfies}$$

$$y'' + y' - 2y = \sin x.$$

Recall $\displaystyle f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

For Δx small then $f'(x)\Delta x \approx f(x+\Delta x) - f(x)$

* change x by little bit, $f(x)$ changes by $f'(x)\Delta x$.

Chain Rule

Problem Most functions we encounter are not simple like e^x or $\cos x$ but are compositions

Ex $\sqrt{x^2+1}$, $e^{\cos x}$, $\ln(\sin x)$

Setup $y = f(u)$ $\quad u = g(x)$

Ex $\quad y = \cos(x^3+1)$ $\qquad f(u) = \cos u$
$\qquad u = x^3+1$

What is $\frac{dy}{dx}$.

Answer: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

Motivation Change x by little bit Δx then

u changes by $\frac{du}{dx} \Delta x$ so

y changes by $\frac{dy}{du} \cdot \frac{du}{dx} \Delta x$

Thm (Chain Rule) Suppose g is diff'ble at x and f is diff'ble at $g(x)$. Then $F(x) = f(g(x))$ is diff'ble at x and

$$F'(x) = f'(g(x))\, g'(x)$$

<u>Notation</u> If $y = f(u)$ $u = g(x)$ Then

$$\frac{dy}{dx} = \frac{dy}{dy} \cdot \frac{dy}{du}$$

<u>EX</u> $y = \cos(x^3+1)$

$g(x) = x^3 + 1$ $\qquad y = f(g(x))$

$f(x) = \cos(x)$

$f'(x) = -\sin x \qquad g'(x) = 3x^2$

$y' = f'(g(x)) \cdot g'(x) = -\sin(x^3+1)(3x^2)$

<u>EX</u> $F(x) = e^{x^2}$. Find $F'(x)$

$f(x) = e^x \qquad g(x) = x^3$

$f'(x) = e^x \qquad g'(x) = 2x$

$F'(x) = f'(g(x)) \cdot g'(x) = e^{x^2} \cdot 2x$

<u>EX</u> $F(x) = \cos(xe^x)$

$F'(x) = -\sin(xe^x)\left[e^x + xe^x\right]$