Recall

1. Tangent line to $y = f(x)$ at point $(a, f(a))$ is $y - f(a) = f'(a)(x - a)$

$$y = f(a) + f'(a)(x - a)$$

Def. $L(x) = f(a) + f'(a)(x - a)$ is the linear approximation or tangent line approximation to $f(x)$ at $x = a$.

Idea. For $x$ near $a$, $L(x) \approx f(x)$

Ex. Approximate $\sqrt{8.99}$ using linear approximation. Is your answer an over or underestimate?

$f(x) = \sqrt{x}$, use $a = 9$

$$f(x) = \frac{1}{2\sqrt{x}} \quad f'(x) = \frac{1}{2} \quad f'(9) = \frac{1}{6}$$

$$L(x) = 3 + \frac{1}{6}(x - 9)$$

$$L(8.99) = 3 + \frac{1}{6}(-0.01) = 2.99833\bar{3}$$

$$\sqrt{8.99} = 2.99833\bar{3}$$

Tangent line is above curve so overestimate.
Example: Use linear approximation to estimate \( \cos(2\pi^\circ) \)

\[ f(x) = \cos x \quad a = \pi / 6 \quad f(a) = \sqrt{3}/2 \]
\[ f'(x) = -\sin x \quad f'(a) = -\sqrt{3}/2 \]

\[ L(x) = \frac{\sqrt{3}}{2} + \frac{-\sqrt{3}}{2}(x - \pi / 6) \]

\[ L \left( \frac{2\pi}{180} \right) = \frac{\sqrt{3}}{2} + \frac{-1}{2} \left( -\frac{\pi}{10} \right) = \frac{\sqrt{3}}{2} + \frac{\pi}{360} = 0.874752 \]

\[ \cos(2\pi) = 0.874619 \]

**Differentials**

Given \( y = f(x) \) the differential is

\[ dy = f'(x) \, dx \]

For \( \Delta x \) small, \( dy \approx \Delta y \)
Ex  \( V = \frac{4}{3} \pi r^3 \)  Volume of sphere

\[ \frac{dV}{dr} = 4\pi r^2 \]  diff.

* for \( dr \) small a change in \( r \) by \( dr \) gives change in \( V \) by about \( 4\pi r^2 \) \( dr \)

+ S.A = 4\pi r^2.  Is this a coincidence?

Review for Exam