Lecture 21

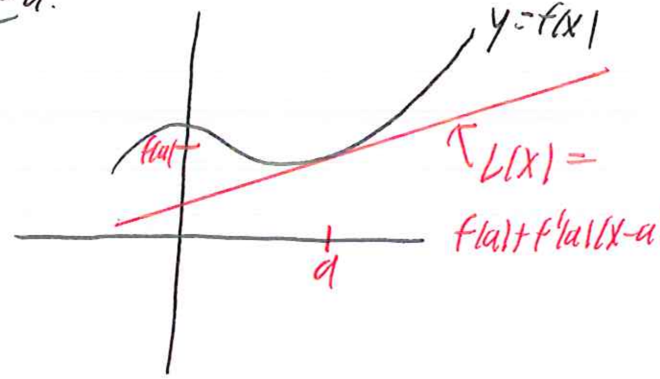
Recall

1. Tangent line to $y=f(x)$ at point $(a, f(a))$ is $y-f(a)=f'(a)(x-a)$

$$y=f(a)+f'(a)(x-a)$$

Def $L(x)=f(a)+f'(a)(x-a)$ is the linear approximation or tangent line approximation to $f(x)$ at $x=a$.

Idea For x near a, $L(x) \approx f(x)$



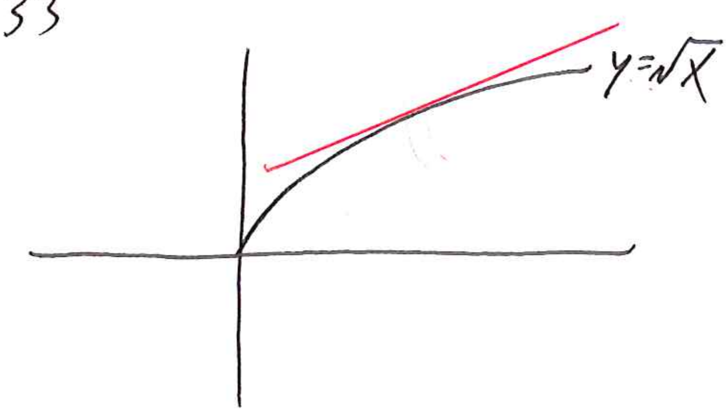
Ex Approximate $\sqrt{8.99}$ using lin. approx. Is your answer an over or underestimate?

$f(x)=\sqrt{x}$, use $a=9$ $\quad f'(x)=\frac{1}{2\sqrt{x}}$ $\quad f'(9)=\frac{1}{6}$ $\quad f(9)=3$

$$L(x)=3+\frac{1}{6}(x-9)$$

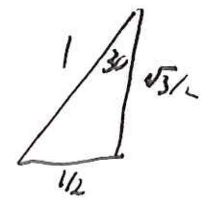
$$L(8.99)=3+\frac{1}{6}(-.01)=2.998\overline{33}$$

$$\sqrt{8.99}=2.9983328$$



tangent line is above curve so overestimate.

Ex Use linear approx to estimate $\cos(29^\circ)$



$f(x) = \cos x \quad a = \pi/6 \qquad f(a) = \sqrt{3}/2$

$f'(x) = -\sin x \qquad\qquad f'(a) = -1/2$

$$L(x) = \frac{\sqrt{3}}{2} + -\frac{1}{2}(x - \pi/6)$$

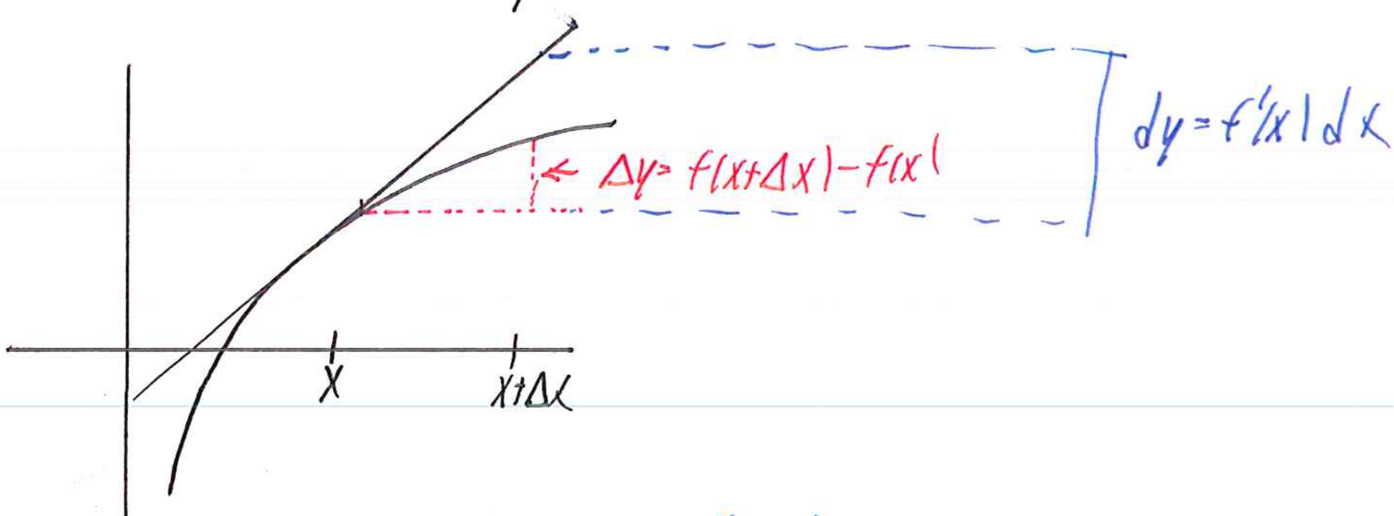
$$L\left(\frac{29\pi}{180}\right) = \frac{\sqrt{3}}{2} - \frac{1}{2}\left(-\pi/180\right) = \frac{\sqrt{3}}{2} + \frac{\pi}{360} = .874752$$

$$\cos(29) = .874619$$

Differentials

Given $y = f(x)$ the differential is

$$dy = f'(x)\,dx$$



For Δx small, $dy \approx \Delta y$

Ex $V = \frac{4}{3}\pi r^3$ volume of sphere

$dV = 4\pi r^2 dr$ diff.

* for dr small a change in r by dr gives change in V by about $4\pi r^2 dr$

* $S.A = 4\pi r^2$ Is this a coincidence?

Review for Exam