Review
1. $f'(x) > 0$ on interval $\Rightarrow$ $f$ is increasing on interval
2. $f'(x) < 0$ on interval $\Rightarrow$ $f$ is decreasing on interval

Ex $f(x) = \frac{1}{2} x^4 - 4x^2 + 3$

$f'(x) = 2x^3 - 8x = 2x(x^2 - 4) = 2x(x - 2)(x + 2)$

Set $f' = 0$ $\Rightarrow \frac{-2}{0} \frac{2}{1}$

Decreasing $(-\infty, -2) \cup (0, 2)$
Increasing $(-2, 0) \cup (2, \infty)$

Ex $f(x) = \frac{e^x}{1 - e^x}$

$f'(x) = \frac{(1 - e^x)e^x - e^x(e^x - e^x)}{(1 - e^x)^2} = \frac{1 - e^{2x}}{(1 - e^x)^2}$

Remark $f'(x)$ can change signs if $f'' = 0$ or is undefined

$1 - e^x = 0 \Rightarrow x = 0$

Increasing $(-\infty, 0) \cup (0, \infty)$
**First Derivative Test**

Suppose $c$ is a critical number of $f(x)$.

1. If $f'(x)$ changes sign from negative to positive at $c$, then $f(c)$ is a local minimum.
2. If $f'(x)$ changes sign from positive to negative at $c$, then $f(c)$ is a local maximum.
3. Else neither.

**Example**

$f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$.

Find local max / min

$f'(x) = \cos x - \sin x$

$0 = f'(x) \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

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<th>$f'(x)$</th>
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<td>$0$</td>
<td>$\frac{\pi}{4}$</td>
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Local max at $(\frac{\pi}{4}, \sqrt{2})$; local min at $(\frac{5\pi}{4}, -\sqrt{2})$.

**Question:** What does $f'(x)$ tell us about the graph?

**Concavity**

- **Concave Up**: '$U'$ shape; graph lies above tangent.
- **Concave Down**: graph lies below tangent.

$f'$ is increasing. $f'$ is decreasing.
Concavity Test

1. If \( f''(x) > 0 \) on an interval then \( f(x) \) is concave up on interval.

2. If \( f''(x) < 0 \) then \( f(x) \) is concave down.

Y possibilities:

<table>
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<tr>
<th>( f' )</th>
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Def. Point \( P \) on \( y = f(x) \) is an inflection point if concavity changes there.

Second Derivative Test:

1. If \( f'(c) = 0 \) and \( f''(c) > 0 \) then local min at \( x = c \)

2. If \( f'(c) = 0 \) and \( f''(c) < 0 \) then local max at \( x = c \)

3. If \( f'(c) = 0 \) and \( f''(c) = 0 \) No inflection point.

Ex: \( y = x^3 \) and \( y = x \) all have \( f'(c) = f''(c) = 0 \).
Ex \( y = x^4 - 4x^3 \) Do every thing

Ex \( y = x^3 \ln x \)

Ex \( y = \frac{x^2}{x-1} \). Classify local max/min w/ both tests

Ex \( y = x^2 - x - \ln x \)