Lecture 28

Review **L'Hospital's Rule** · Handles limits of the "$\frac{0}{0}$ or $\frac{\infty}{\infty}$"

1. Types "$0 \cdot \infty$" handle by writing as quotient $f \cdot g = \frac{f}{1/g} = \frac{g}{1/f}$

2. Types $\infty - \infty$: Try to make a quotient.

Ex $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right) = \lim_{x \to 0^+} \frac{e^x - 1 - x}{x(e^x - 1)}$ type $\frac{0}{0}$

$$\text{LHR} = \lim_{x \to 0^+} \frac{e^x - 1}{xe^x + e^x - 1} \overset{\text{LHR}}{=} \lim_{x \to 0^+} \frac{e^x}{xe^x + e^x + e^x} = \frac{1}{1}$$

$= 1$

3. Types 0^0, ∞^0, 1^∞

- take ln 1st
- proceed as before
- do "e^-"

Ex $\lim_{x \to \infty} x^{1/x}$

$$y = x^{1/x}$$

$$\ln y = \frac{\ln x}{x}$$

$$\lim_{x \to \infty} \ln y = \lim_{x \to \infty} \frac{\ln x}{x} \overset{\text{LHR}}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = 0$$

So $\lim_{x \to \infty} y = e^0 = 1$

4. Nonexample: $\lim_{h \to 0} \frac{\sinh}{h} = \lim_{h \to 0} \frac{\cosh}{1} = 1$

* Why is this cheating?

· We used this limit to find $\frac{d}{dx}(\sin x)$!

Curve Sketching

Goal Sketch graph $y = f(x)$ using everything we know

1. Domain
2. x & y intercepts
3. Symmetry ($f(x) = f(-x)$ even function, graph symmetric about y axis !
   $f(x) = -f(-x)$ odd function, " " " origin
4. Horizontal & Vertical asymptotes

   $\lim_{x \to \pm\infty} f(x)$

   1. look for div. by 0
   2. Plug in nearby points to get behavior.
5. increase/decrease (using f'), local max/mins
6. concavity & inflection points

   $\rightsquigarrow$ sketch

Ex $f(x) = \dfrac{(x+1)^2}{1+x^2}$

1. Domain $(-\infty, \infty)$
2. Intercepts $(0,1)$ $(-1,0)$
3. No symm.
4. H.A. $\cancel{x} = 1$ both dir.

$$f'(x) = \frac{2(1-x^2)}{(1+x^2)^2}$$

f': $-$ on $(-\infty,-1)$, $+$ on $(-1,1)$, $-$ on $(1,\infty)$

decreasing $(-\infty,-1) \cup (1,\infty)$

incr $(-1,1)$

local min $(-1,0)$

local max $(1,2)$

$$f''(x) = \frac{4x(x^2-3)}{(1+x^2)^3}$$

f'': $-$ on $(-\infty,-\sqrt{3})$, $+$ on $(-\sqrt{3},0)$, $-$ on $(0,\sqrt{3})$, $+$ on $(\sqrt{3},\infty)$

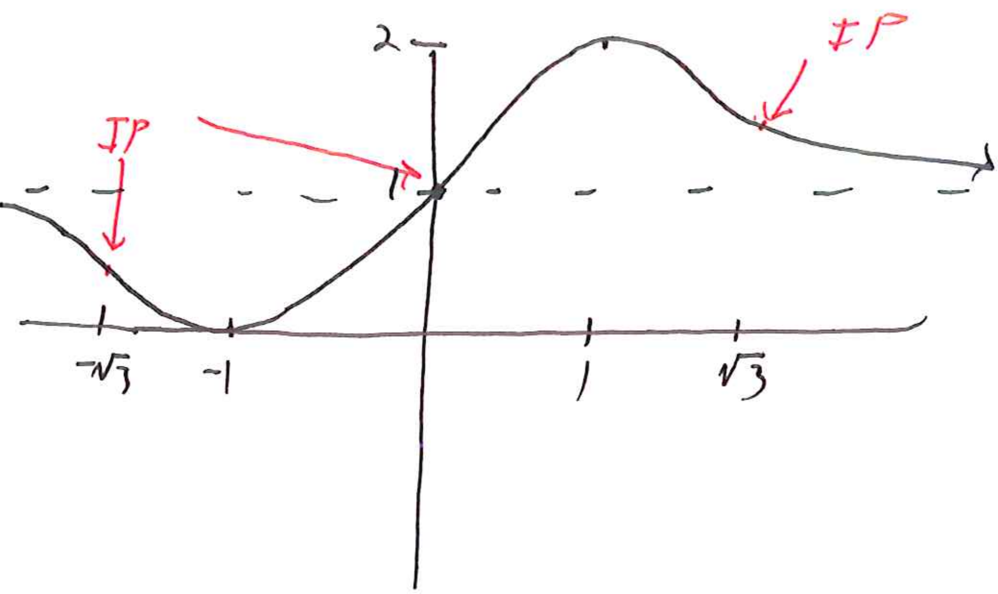
concave down $(-\infty,-\sqrt{3}) \cup (0,\sqrt{3})$

up $(-\sqrt{3},0) \cup (\sqrt{3},\infty)$

inf. point $\left(-\sqrt{3}, \frac{(1-\sqrt{3})^2}{4}\right)$

$(0,1)$

$\left(\sqrt{3}, \frac{(1+\sqrt{3})^2}{4}\right)$



$y = \dfrac{x}{x^3-1}$ H.A $y=0$

V.A. $x=1$

$\lim_{x\to 1^+} f(x) = \infty$

$\lim_{x\to 1^-} f(x) = -\infty$

Intercept: $(0,0)$

$y' = \dfrac{(x^3-1)-3x^3}{(x^3-1)^2} = \dfrac{-1-2x^3}{(x^3-1)^2}$

$2x^3 = -1$

$x^3 = -1/2$

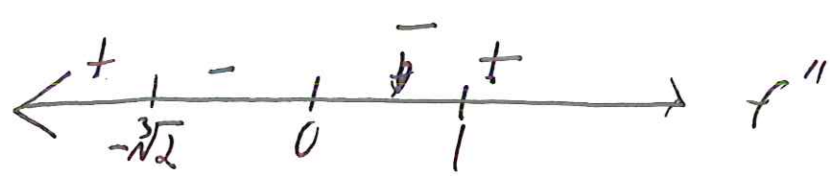
$x = \dfrac{-1}{\sqrt[3]{2}}$

local max $x = -\frac{1}{\sqrt[3]{2}}$

increasing $(-\infty, -\frac{1}{\sqrt[3]{2}})$

decreasing $(-\frac{1}{\sqrt[3]{2}}, 1) \cup (1, \infty)$

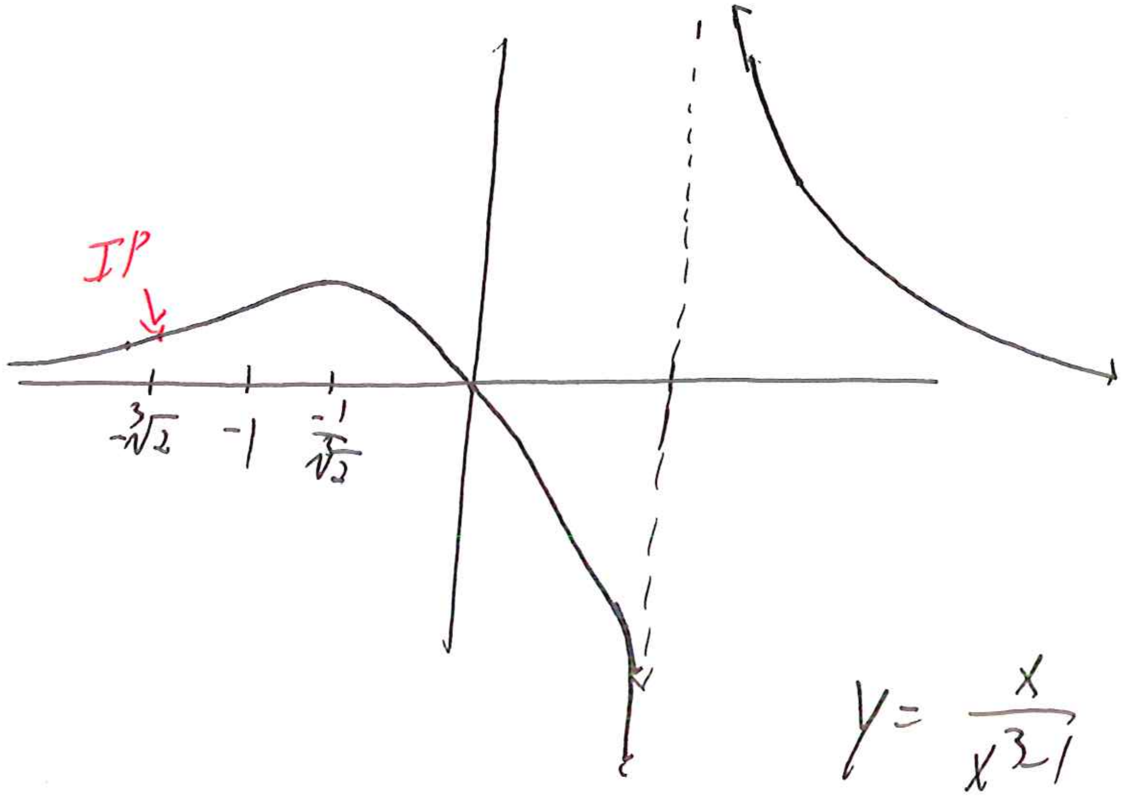
$y'' = \dfrac{6x^2(x^3+2)}{(x^3-1)^3}$



concave up $(-\infty, -\sqrt[3]{2}) \cup (0, \infty)$

down $(-\sqrt[3]{2}, 0) \cup (0, 1)$

I.P. $\left(-\sqrt[3]{2}, \dfrac{-\sqrt[3]{2}}{-3}\right)$



$$y = \frac{x}{x^3 - 1}$$