Lecture 31 - Antiderivatives

Recall Thm: If $f'(x) = g'(x)$ on I then $f(x) = g(x) + C$.

Def $F(x)$ is an antiderivative of $f(x)$ on I if

$$F'(x) = f(x) \text{ on } I.$$

Ex $\frac{x^3}{3}$ is an anti. der. of x^2.

Rmk If $F(x)$ is an antiderivative of $f(x)$, the most general one is $F(x) + C$.

Problems

Find most general antiderivatives:

1. $f(x) = 2x + 3$ $\quad F(x) = x^2 + 3x + C$

2. $f(x) = \sec^2 x$ $\quad F(x) = \tan x + C$

3. $f(t) = \frac{1 + t + t^3}{\sqrt{t}} = t^{-1/2} + t^{1/2} + t^{3/2}$ $\quad F(t) = 2t^{1/2} + \frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2} + C$

4. $f(x) = 3^x$ $\quad F(x) = \frac{3^x}{\ln 3} + C$

5. $f(x) = e^3$ $\quad F(x) = e^3 x + C$

Rmk $\frac{d}{dx}(\ln x) = \frac{1}{x}$ if $x > 0$

$\frac{d}{dx}(\ln|-x|) = \frac{1}{x}$ if $x < 0$

* Antiderivative of $\frac{1}{x}$ is $\ln|x| + C$.

Initial value problems

1. Suppose $f''(x) = 8x^3 + 5$ $f(1) = 0$ $f'(1) = 8$. Find $f(x)$!

$f'(x) = 2x^4 + 5x + C$ $\quad 8 = 2 + 5 + C \rightarrow C = 1$

$f'(x) = 2x^4 + 5x + 1$

$f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x + C$ $\quad 0 = \frac{2}{5} + \frac{5}{2} + 1 + C$

$C = -3.9$

$$\boxed{f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x - 3.9}$$

2. Particle moves with $v(t) = \sin t - \cos t$ $s(0) = 0$. Find $s(t)$!

$s(t) = -\cos t - \sin t + C$ $\quad \boxed{s(t) = -\cos t - \sin t + 1}$

$0 = -1 + C \rightarrow C = 1$

3. Ball thrown upward 48 ft/sec off cliff 432 ft high.

- Find formula for $h(t)$
- Find max ht.
- When hit ground

Rmk $a(t) = \frac{dv}{dt} = -32$ ft/sec^2

Given $v(0) = 48$ $\quad s(0) = 432$

$v(t) = -32t + C$ $\quad v(t) = -32t + 48$

$s(t) = -16t^2 + 48t + C$ $\quad s(t) = -16t^2 + 48t + 432$.

Max ht when $s'(t) = 0$ $\quad t = 1.5$ $\quad \cdot\ s(1.5) = 468$

$\cdot\ s(t) = 0 \rightarrow t = \frac{3}{2} \pm \frac{3}{2}\sqrt{13}$ $\quad -3.908,\ 6.908$

More generally $s(t) = -16t^2 + v(0)t + s(0)$ in ft

$= -4.9t^2 + v(0)t + s(0)$ in m.

Problems

1. Stone dropped off cliff. Hits ground at 120 ft/sec. How high was cliff.

2. $f''(x) = e^x - 2\sin x$, $f(0) = 3$, $f(\pi/2) = 0$. Find $f(x)$.

3. $a(t) = t^2 - 4t + 6$ $s(0) = 0$ $s(1) = 20$. Find $s(t)$.

4. Car brakes w/ constant deceleration 16 ft/sec^2 stops after 200 ft skid mark. How fast was it going when brakes were applied?

$a(t) = -16$

$v(t) = -8t + v(0)$

$s(t) = -4t^2 + v(0)t + s(0)$

Assume $s(0) = 0$. $s(t) = -4t^2 + v(0)t$.

$s(t) = 200$ when $v(t) = 0$, i.e. $t = v(0)/8$

$$200 = -4\left(\frac{v(0)}{8}\right)^2 + \frac{v(0)^2}{8}$$

$$= -\frac{v(0)^2}{16} + \frac{v(0)^2}{8} = \frac{v(0)^2}{16} \rightarrow v(0)^2 = 3200$$

$v(0) = 56.56$ ft/sec.