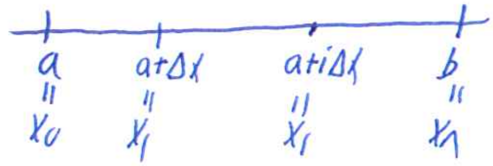
Lecture 34

Recall



1. divide $[a,b]$ into n pieces width $\Delta x = \frac{b-a}{n}$

2. Choose $x_i^* \in [x_{i-1}, x_i]$ sample point.

3. Calculate "signed area" $f(x_i^*)\Delta x$ and add up $\sum_{i=1}^{n} f(x_i^*)\Delta x$

Def $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$ <u>definite integral</u>.

* Basic properties of Σ give properties of $\int$

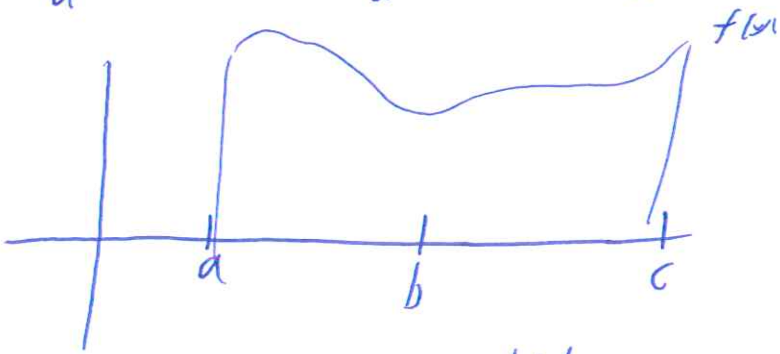
Ex $\sum_{i=1}^{n} f(x_i)^* + g(x_i)^* = f(x_1) + g(x_1) + f(x_2) + g(x_2) + \cdots + f(x_n) + g(x_n)$

$= f(x_1) + \cdots + f(x_n) + g(x_1) + \cdots + g(x_n)$

$= \sum_{i=1}^{n} f(x_i^*) + \sum_{i=1}^{n} g(x_i^*)$

so $\int_a^b f+g = \int_a^b f + \int_a^b g$

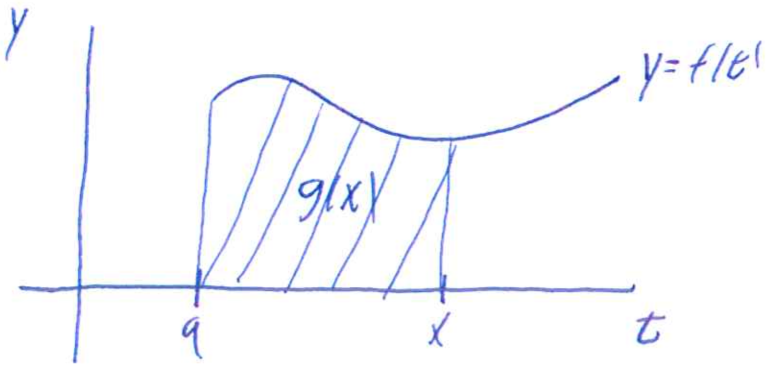
Ex $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$



Also true if b not middle.

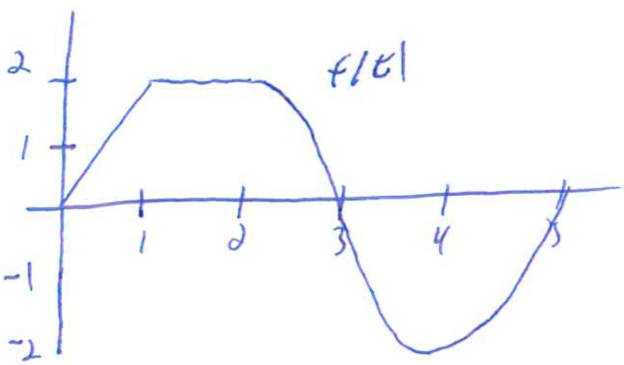
Fundamental Thm of Calculus

Consider $g(x) = \int_a^x f(t)\,dt$, where $f(t)$ is continuous



$g(x)$ is "area so far"

EX



$g(1) = 1$

$g(2) = 3$

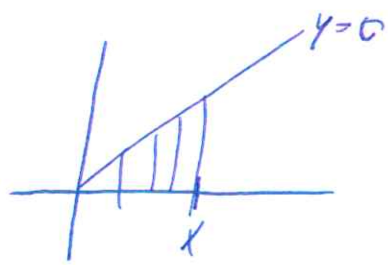
$g(3) \approx 4.8$

$g(4) \approx 4.8 - 1.8 = 3$

$g(5) \approx 3 - 1.8 = 1.2$

EX $f(t) = t$

$$g(x) = \int_0^x t\,dt$$

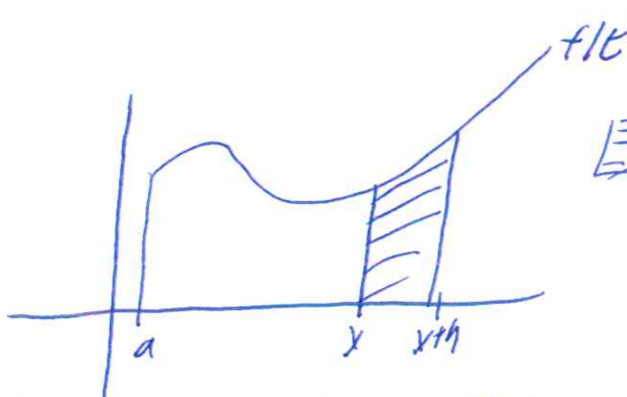


$g(x) = \frac{1}{2} x^2$

so $g' = f$!

Problem What is $g'(x)$?

$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$



$= g(x+h) - g(x)$

as $h \to 0$ looks like rectangle, base h, height $f(x)$

Area $f(x)\,h$

FTOC Part I Suppose f is continuous on $[a,b]$. Let

$$g(x) = \int_a^x f(t)\,dt \qquad a \le x \le b$$

Then $g(x)$ is continuous on $[a,b]$, diff'ble on (a,b) and

$$g'(x) = f(x).$$

Alternate Notation: $$\boxed{\frac{d}{dx}\int_a^x f(t)\,dt = f(x)\,!} \quad (*)$$

Remarks

1. Changing a changes $g(x)$ by a constant, so does not change $g'(x)$.

   This is why "a" does not appear in right side of $(*)$

2. Every continuous function has an antiderivative!
   - may not be an elementary function
   - computer can still calculate it as accurately as we wish.

Ex xe^{x^2} has anti-deriv $\frac{1}{2}e^{x^2}$.

Does $f(x) = e^{x^2}$ have one?

Yes! $\frac{d}{dx} \int_a^x e^{t^2} dt = e^{x^2}$

Ex Fresnel Function

$$S(x) = \int_0^x \sin(\pi t^2/2)\, dt$$

so $S'(x) = \sin(\pi x^2/2)$

Suppose $F'(x) = f(x)$, so $F(x)$ an antider. of $f(x)$.

But so is $\int_a^x f(t)\,dt$! So

$$F(x) - \int_a^x f(t)\,dt = C \quad \text{plug in } x=a \quad F(a) = C$$

So $F(x) = \int_a^x f(t)\,dt + F(a)$ plug in $x=b$

$$F(b) - F(a) = \int_a^b f(t)\,dt$$

FTOC Part 2 Suppose $F'(x) = f(x)$ on $[a,b]$

Then $\int_a^b f(x)\,dx = F(b) - F(a)$!