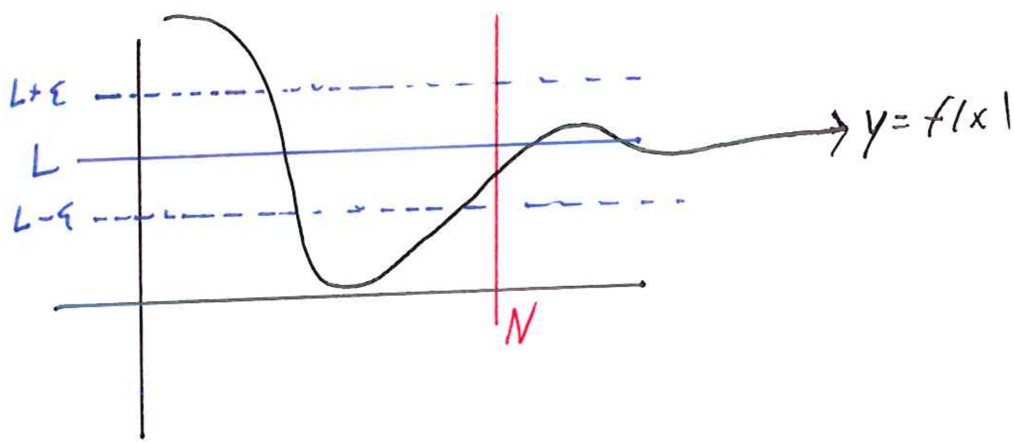
Lecture 7 Review Int. Value Thm

Ex
- Prove $\cos x = x^3$ has a solution
- Non continuous ex.

Limits at Infinity

Def Suppose $f(x)$ is defined on (c,∞) for some c. Say $\lim_{x\to\infty} f(x) = L$

if for any $\varepsilon > 0$ there is an N so if $x > N$ then $|f(x) - L| < \varepsilon$.



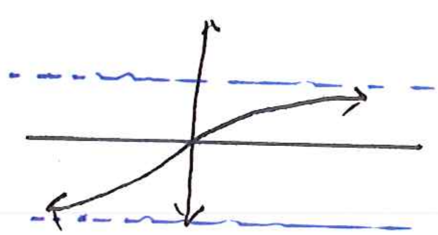
Rmk Corresponding def for $\lim_{x\to -\infty} f(x) = L$.

Def Say graph of $y = f(x)$ has a horizontal asymptote $y = L$ if

$\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to -\infty} f(x) = L$.

Immediate: At most two horizontal asymptotes

Examples
1. $y = 1/x$
2. $y = \tan^{-1} x$



$\lim_{x\to\infty} \tan^{-1} x = \pi/2$

$\lim_{x\to -\infty} \tan^{-1} x = -\pi/2$

3. $\lim_{x\to\infty} \frac{\sin x}{x} = 0$ * graph crosses asymptote infinitely many times

4. r - rational, $\lim_{x\to\infty} 1/x^r = 0$

5. $\lim_{x\to-\infty} \frac{6x^3+2x+1}{7x^3-x}$

6. Find all hor & vert asym. for $f(x) = \frac{\sqrt{6x^2+1}}{5x-3}$

Warning: When $x>0$ then $\sqrt{x^2} = x$
When $x<0$ then $\sqrt{x^2} = -x$

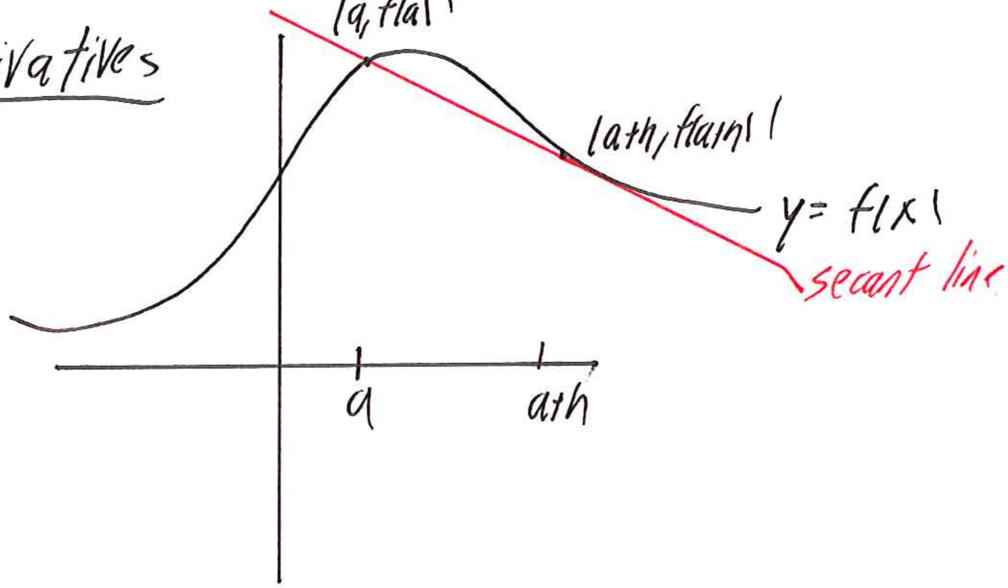
7. $\lim_{x\to\infty} \left(\sqrt{x^2+3} - x\right)$ "$=\infty-\infty$" anything is possible

8. $\lim_{x\to\infty} e^x$, $\lim_{x\to-\infty} e^x$ • "infinite limit at ∞" you figure out def

9. Sketch a graph w/ $\lim_{x\to 2} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = 3$ $\lim_{x\to-\infty} f(x) = -1$

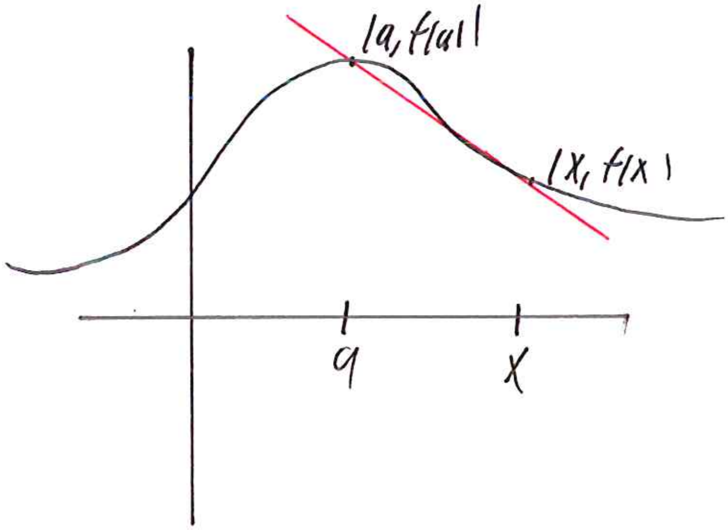
$\lim_{x\to -3^+} f = -\infty$ $\lim_{x\to -3^-} f = \infty$ $f(-3) = 0$

Derivatives



Key point $\dfrac{f(a+h)-f(a)}{h}$ is slope of secant line.

a.k.a. avg rate of change of $f(x)$ with respect to x from $x=a$ to $x=a+h$



slope is $\dfrac{f(x)-f(a)}{x-a}$

Def The derivative of $f(x)$ at $x=a$ is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

if it exists

$$= \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

Interpretation $f'(a) =$ slope of tangent line at $(a, f(a))$

$f'(a) =$ instantaneous rate of change

Ex $f(x) = x^2$. Find $f'(3)$. Find tang line at $(3, 9)$.

Ex $f(x) = 1/\sqrt{x}$. Find $f'(a)$.