Math 353 Fall 2016 Midterm Exam Review Sheet

The exam will cover from Chapter 2-7.2 plus other material done in class.

Definitions: You should know each of these definitions and for various types of numbers, you should be able to calculate them in small examples:

- P(n,r)=n!/(n-r)!- the number of ways to make an ordered list of length r from an n element set.
- C(n,r)- The binomial coefficient, equal to the number of r element subsets of an n element set.
- Multinomial coefficients.
- S(n,r)- Stirling number of the 2nd kind, the number of ways to put n distinct balls in r identical boxes with no empty boxes.
- s(n,r)- Stirling number of the 1st kind, the coefficient of x^r in $[x]_n$.
- p(n,r)- The number of permutations in S_n with exactly r cycles (=s(n,r) by theorem)
- Derangements
- C_n- the Catalan numbers, including the various objects they count, Catlan sequences, Dyck paths, triangulations, expressions, 231-avoiding permutations, etc..
- p(n), $p_k(n)$,
- Ferrer's diagrams of partitions, conjugate of a partition, self-conjugate partitions..
- Tableau and standard tableau
- Permutations avoiding a certain pattern (e.g. 231 on the homework)
- Symmetric polynomials, Schur polynomials.
- Generating function of a sequence
- Euler phi function, formula for it form I/E principle.

Counting Problems:

- Counting and probability problems using principle of multiplication of choice, C(n,r) and P(n,r), knowing when each applies (does order matter, overcounting, etc…). The binomial theorem. (for example Ex: 2.2.1-2.2.3, 2.3.1- 2.3.3, all exercises in 2.4) Poker/bridge type problems.

- Counting the number of permutations in S_n with a given cycle structure.

• 8 different “occupancy problems” in Table 3.1, know when to apply each.

• Simple combinatorial proofs using binomial coefficients (like 2.3.3B, 2.3.4A)

Other material

• You should know the recurrence relations for C(n,r), S(n,r) and s(n,r) and combinatorial explanations for the first two. Pascal’s triangle.

• Understand how S(n,r) and s(n,r) are change of basis coefficients between two natural bases for polynomials of degree n with no constant term.

• Know the inclusion/exclusion principle and apply it in counting problems. You do not need to know the formula on the top of p.56 but you should be able to figure it out in small examples using I/E principle. For example the argument on p.55 you should be able to replicate, you should be able to solve Problem 4.3.

• Same thing for the formula for the number of derangements.

• Easy combinatorial proofs involving partitions, bijections with Ferrer’s diagrams.

• Be able to do the row insertion algorithm and the Robinson-Schensted-Knuth algorithm.

• Write down generating functions for some simple sequences.

• Recurrence relation satisfied by the Catalan numbers and how it is applied to show they count 231-avoiding permutations and triangulations.

REVIEW HOMEWORK PROBLEMS!