Math 353 Homework #11- Due Monday 12/5/16

1. Use Polya's theorem to compute the number of 5×5 chessboards with 10 red squares, 12 blue squares and 3 green squares, up to symmetry. You will likely need to use a computer algebra system like Maple in the final step.

2. Taking rotational symmetries into account, how many ways are there to color the vertices of a cube so that four are blue, two are red and two are green?

3. Recall that the integer lattice $\mathbb{Z}^3$ consists of all points $(a_1, a_2, a_3) \in \mathbb{R}^3$ such that a_1, a_2, a_3 are integers. Suppose we choose 9 distinct points in $\mathbb{Z}^3$. Prove the line segment between some two of the 9 points contains another point in $\mathbb{Z}^3$. Hint: You can actually show the "another point" may be chosen to be the midpoint.

4. Show that given any 9 distinct natural numbers it is possible to chose 5 whose sum is divisible by 5.

5. 15.3.4B