Math 353 Homework #1- Due Monday 9/12/16

1. Suppose a biased coin comes up heads 65 percent of the time. If we flip the coin 10 times, what is the probability we will get **at least** 8 heads?

$$\binom{10}{8}(.65^8)(.35^2) + \binom{10}{9}(.65^9)(.35^1) + \binom{10}{10}(.65^{10}).$$

2. 2.5.2B Three letters occur once, three occur twice and two occur three times. So by the multinomial theorem we want the multinomial coefficient:

$$\binom{15}{2,2,2,3,3}$$

which is 4540536000.

3. 2.6.2B The number of permutations with cycle type $(4,3,2,1)$ is

$$\frac{10!}{4 \cdot 3 \cdot 2 \cdot 1} = 151,200.$$

For cycle type $(4,2,2,2)$ it is :

$$\frac{10!}{4 \cdot 2^3 \cdot 3!} = 19800.$$

4. 2.6.3B Let $d(n)$ be the number of derangements in S_n (We will derive a formula eventually. Then the

number of permutations with exactly one fixed point can be calculated by first choosing the fixed point (n choices) and then permuting the remaining numbers with no fixed points ($d(n-1)$ choices.) So the probability is

$$\frac{nd(n-1)}{n!} = \frac{d(n-1)}{(n-1)!}.$$

5. Find the probability that a bridge hand has 6-3-2-2 distribution. Show your work.

There are $\binom{52}{13}$ bridge hands. To make one with 6-3-2-2 distribution we can choose the 6-card suit (4 choices), the 3 card suit (2-choices) and then choose the six cards ($\binom{13}{6}$ choices), the 3 cards ($\binom{13}{3}$ choices) and the two cards twice ($\binom{13}{2}^2$ choices). So the final probability is:

$$\frac{4 \cdot 3 \cdot \binom{13}{6}\binom{13}{3}\binom{13}{2}\binom{13}{2}}{\binom{52}{13}} = .05642\ldots$$

6. In class we worked out the odds that the opponents missing 4 cards in a suit split 4-0, 3-1 or 2-2 (problem 2.4.4B). Complete the same calculation when we have an 8-card fit, i.e. the opponents have 5 cards. What are the odds of a 5-0, 4-1, or 3-2 split?

For a 5-0 split with 5 cards on our left the odds are

$$\frac{\binom{5}{5}\binom{21}{8}}{\binom{26}{13}} = .0196$$

so the probability of 5-0 or 0-6 is .0391.

For a 4-1 split with 4 cards on our left the odds are

$$\frac{\binom{5}{4}\binom{21}{9}}{\binom{26}{13}} = .1413$$

so the probability of 4-1 or 1-4 is .2826.

For a 3-2 split with 3 cards on our left the odds are

$$\frac{\binom{5}{3}\binom{21}{10}}{\binom{26}{13}} = .339$$

so the probability of 3-2 or 2-3 is .6782.