Name: SOLUTIONS

Math 353 Midterm Exam - October 17, 2014

Instructions: You may not use any notes, books, calculators, etc... It is ok if your final answers include binomial coefficients and if you do not multiply out exponentials.

1. (55 points) Short answer, little or partial credit.

a. Define a derangement.

A permutation with No fixed points, i.e. o(i) \$\neq i\$.

b. A class of 20 students wishes to elect a president, vice president, and three senators. How many ways can this be done?

20.19. (18)

c. A five-card poker hand is dealt. What is the probability of getting two-pair? (for example AATT3 is two-pair but AAATT is not, it is a full-house.)

 $\frac{\binom{13}{2}\binom{4}{2}\binom{4}{2}\cdot 44}{\binom{52}{5}}$

d. Let $\sigma = (123)(45)(678)$ and $\tau = (1568)(234)(7)$ be elements of the symmetric group S_8 .. Write the products $\sigma\tau$ and σ^2 in cycle notation.

or= (1435782)(6)

o= (132)/687)(41/5)

e. Let $S(n,k)$	denote the Stirling	number of	the second	kind.	Calculate	S(5)	, 2)).

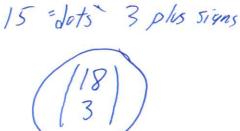
5 distinct objects, 2 boxes, No empties

4+1 5 choices

3+2 (5)=10 chaires



f. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 15$ where the x_i are nonnegative integers?



g. How many triangulations are there of a octagon?

 $C_6 = {\binom{12}{a}}_7 = \frac{\frac{1}{12} \frac{3}{12} \frac{3}{$



h. Calculate p(8,3), the number of partitions of 8 with less than or equal to three parts.

8 7/62

611

53

521

44

421

332

i. A fair coin is tossed 6 times. What is the probability of getting at least 4 heads?

$$\frac{\binom{6}{4}}{2^6} + \frac{\binom{6}{5}}{2^6} + \frac{\binom{6}{6}}{2^6} = \frac{22}{64} = \frac{11}{32}$$

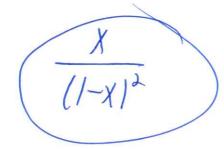
j. A biased coin comes up heads with probability 0.8. If it is tossed 6 times what is the probability of getting at least 4 heads.

4 heads: [4](.84](2) 5 heads [5](.85)(2)
6 heads [6](.84)

(6/1.87/1.22) + (5/1.85/1.2) + (6/1.86)

k. Find the generating function for the sequence $a_n = n$.

$$\frac{1}{1} = 1 + \lambda x + 3 x^2 + 4 x^3$$



2. (15 points) A bag contains 3 red, 2 blue and 6 green marbles. A marble is sampled and replaced 7 times. Find the probability that all three colors have already been seen.

Let MR = # ways to have No reds Similarly MB, Mc. Total of 11 draws, need to subtact IMRVMsVMal. | MR VMB VMG |= | MR | + | MB | + | MG | - | MR MB | - | MR MG | - | MD MG) + /MONMRAMA = 8+97+57-67-27-37+0

- 3. (15 points) a. Define the Stirling number of the second kind, S(n,k).
 - b. Give a combinatorial proof of the identity: $S(n+1,k+1) = \sum_{i=k}^{n} {n \choose i} S(i,k)$.
- a. The # of ways to assign n distinct balls into Kidentical boxes with no empty boxes.

b. Fix a ball b. Divide up the possible assignment based on the # of balls in the remaining k boxes what be There must be at least k since each box has along.

In the # of ways to assign whi balls not in bis box is

[1] Suik

chase bulls assign to the bases
not with b we emptie:

all assignments of nel balls to K+1 boxes are counted once

Thus

S(n+1,K+1)= (2) S(K,K) + (2) S(K+1,K)+...+ (2) S(N,K)

as desired //

4. (15 points) Consider the sequence given by $a_1 = 8$, $a_2 = 46$ and $a_n = 7a_{n-1} - 10a_{n-2}$ for $n \ge 3$. Solve the recursion to find a formula for a_n .

$$Q_{n}-7a_{n-1}+10a_{n+}=0$$

$$Polynomial is \chi^{2}-7x+10 = (x-5)(x-2)$$

$$General solution A.5^{1}+B.2^{n}$$

$$8=A.5+2.B$$

$$46=A.25+4.B$$

$$\Rightarrow 30=15A \Rightarrow A=2$$

$$\Rightarrow B=-1$$

$$Q_{n}=2.5^{n}-1.2^{n}$$