

## Math 417-517 - Fall 2019 - Problems and Announcements

(Problems in Greenberg: Advanced Engineering Mathematics)

1. (Aug 26) review chapters 9 and 10
2. (Aug 28) section 13.3, problems 2(a), 3(a,f).  
Also special problem: Find the tangent plane to  $z = 1/(x^2 + y^2)$  at  $x = 1, y = 2$ . (Answer:  $z = \frac{3}{5} - \frac{2}{25}x - \frac{4}{25}y$ .)
3. (Aug 30) section 13.4, problems 3(a), 4(a), 6(a,d).
4. (Sept 4) section 13.6 problems 2a, 3a, 4a, 5a, 6a, 12.
5. (Sept 9) special problem: The function  $x = u^3 - v^3, y = (u + v)^2$  sends  $(1,0)$  to  $(1,1)$ . Show there is an inverse function  $u = u(x, y), v = v(x, y)$  defined near  $(1,1)$  and find all the partial derivatives at that point. (Answer:  $u_x = \frac{1}{3}, u_y = 0, v_x = -\frac{1}{3}, v_y = \frac{1}{2}$  ).
6. (Sept 11) section 13.7 problems 5(a,g), 10.
7. (Sept 13) section 13.8 problems 1(a,d), 3, 6.
8. (Sept 16) Three problems on calculus of variations:

- (a) Find the function  $y = y(x)$  with  $y(0) = 0, y(1) = 0$  which minimizes the integral

$$I(y) = \int_0^1 ((y')^2 + 12xy)dx$$

What is the minimum value of  $I(y)$ ? (Answer:  $y = x^3 - x, I(y) = -\frac{4}{5}$ )

- (b) A pendulum of mass  $m$  on a string of length  $\ell$  makes an angle  $\theta = \theta(t)$  with the verticle at time  $t$ . The kinetic energy is  $T = \frac{1}{2}m\ell^2\theta'^2$ , the potential energy is  $V = mg\ell(1 - \cos\theta)$ , and the Lagrangian is the difference  $L = T - V$ . To find the motion we seek to minimize the action  $I(\theta) = \int_{t_0}^{t_1} L(\theta, \theta')dt$ . What is the equation of motion we get from this formulation?

(c) Show that the Euler equation can be written in the form

$$\frac{d}{dx}(F - y'F_{y'}) = F_x$$

and conclude that if  $F(x, y, y')$  does not explicitly depend on  $x$  then  $F - y'F_{y'} = \text{constant}$ .

9. (Sept 20)

section 14.3 problems 1(a,h,k)

section 14.4 problems 4(a,d)

section 14.5 problems 1(a,d), 7(a)

10. (Sept 27)

section 14.6 problems 3b, 5d, 9c

11. (Sept 30)

section 15.2 problems 1f, 2a, 3a

12. (Oct 2)

section 15.3 problems 1a, 4a, 12(a,c)

13. (Oct 4)

section 15.4 problems 7, 11(a,g)

14. (Oct 7)

section 15.5 problems 1(a,c), 10(a,c,e), 13(a,d)

15. (Oct 9)

section 15.6 problems 2, 4a

16. (Oct 11)

section 16.4 problems 2d, 4a

section 16.5 problems 1(a,d)

section 16.6 problems 1d, 6f

17. (Oct 14)

section 16.8 problems 1(d,l), 12f (suggestion: start in Cartesian coordinates)

18. (Oct 18)

section 16.9 problems 2e, 3(d,g)

**Exams:**

The first exam is Wednesday, September 25. It covers Chapter 13 (except 13.5), the calculus of variations, and Chapter 14 (except 14.6). Just bring a pencil and eraser. No note cards, calculators, etc.

The second exam is Wednesday, October 23. It covers 14.6, 15, 16.1-16.6, 16.8, 16.9.1. Same restrictions as the first exam.

The third exam is Wednesday, Nov 20.