DUE: Monday, March 23rd, 4pm to box in 244 Mathematics Building

(1) A warm-up; note parts (b)-(d) correspond to 5.1 Problem 1. Let

$$\mathbf{A} = \begin{bmatrix} 2 & -3 \\ 4 & 7 \end{bmatrix} \qquad \text{and} \qquad \mathbf{B} = \begin{bmatrix} 3 & -4 \\ 5 & 1 \end{bmatrix}$$

Compute (a) $\mathbf{A} + \mathbf{B}$; (b) $3\mathbf{A} - 2\mathbf{B}$; (c) $\mathbf{AB}$; (d) $\mathbf{BA}$.

(2) Consider the mass-spring system described in 4.1 Example 1:

$$\begin{aligned} m_1 x_1'' &= -k_1 x_1 + k_2(x_2 - x_1), \\ m_2 x_2'' &= -k_2(x_2 - x_1) + f(t) \end{aligned}$$

By defining $y_1 = x_1, y_2 = x_1', y_3 = x_2, y_4 = x_2'$, transform the system above to an equivalent 4-dimensional first order system.

(3) Define the following vectors:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \qquad \mathbf{y}' = \begin{bmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{bmatrix} \qquad \mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ f(t) \end{bmatrix}$$

Write the matrix $\mathbf{Q}$ for which the answer to (2) is expressed in matrix notation by

$$\mathbf{y}' = \mathbf{Q}\mathbf{y} + \mathbf{f}(t)$$

(4) Consider the two-dimensional first order system of differential equations:

$$\begin{aligned} x_1' &= ax_1 + bx_2 \\ x_2' &= cx_1 + dx_2 \end{aligned}$$

(a) Assuming $b \neq 0$, use the first equation to solve for x_2 in terms of x_1, x_1'.

(b) Derive your answer to (a) to write x_2' in terms of x_1', x_1''.

(c) Substitute your answers for (a) and (b) to obtain a second-order differential equation fulfilled by x_1.

Also 4.2 Problems **2*, 6*, 10** and 5.1 Problems **17, 22, 27, 30, 31, 36, 39, 42.**
*Phase portraits not required for now.

Notice 5.1, 22, 27, and 30 have three parts each: (a) show each $\mathbf{x}_i$ is a solution, (b) show each list of solutions is linearly independent, and (c), write the general solution. To do part (a), you should compute $\mathbf{M}\mathbf{x}_i$ and $\mathbf{x}_i'$ and show they are equal for each i. Here $\mathbf{M}$ is the matrix in the given differential equation $\mathbf{x}' = \mathbf{M}\mathbf{x}$.