Homework #6  Spring 2015

DUE: Monday, March 23rd, 4pm to box in 244 Mathematics Building

(1) A warm-up; note parts (b)-(d) correspond to 5.1 Problem 1. Let

\[ A = \begin{bmatrix} 2 & -3 \\ 4 & 7 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & -4 \\ 5 & 1 \end{bmatrix} \]

Compute (a) \( A + B \); (b) \( 3A - 2B \); (c) \( AB \); (d) \( BA \).

(2) Consider the mass-spring system described in 4.1 Example 1:

\[
\begin{align*}
m_1 x_1'' &= -k_1 x_1 + k_2 (x_2 - x_1), \\
m_2 x_2'' &= -k_2 (x_2 - x_1) + f(t)
\end{align*}
\]

By defining \( y_1 = x_1, y_2 = x_1', y_3 = x_2, y_4 = x_2' \), transform the system above to an equivalent 4-dimensional first order system.

(3) Define the following vectors:

\[ y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}, \quad y' = \begin{bmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{bmatrix}, \quad f(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ f(t) \end{bmatrix} \]

Write the matrix \( Q \) for which the answer to (2) is expressed in matrix notation by

\[ y' = Qy + f(t) \]

(4) Consider the two-dimensional first order system of differential equations:

\[
\begin{align*}
x_1' &= ax_1 + bx_2 \\
x_2' &= cx_1 + dx_2
\end{align*}
\]

(a) Assuming \( b \neq 0 \), use the first equation to solve for \( x_2 \) in terms of \( x_1, x_1' \).

(b) Derive your answer to (a) to write \( x_2' \) in terms of \( x_1', x_1'' \).

(c) Substitute your answers for (a) and (b) to obtain a second-order differential equation fulfilled by \( x_1 \).

Also 4.2 Problems 2*, 6*, 10 and 5.1 Problems 17, 22, 27, 30, 31, 36, 39, 42. *Phase portraits not required for now.

Notice 5.1, 22, 27, and 30 have three parts each: (a) show each \( x_i \) is a solution, (b) show each list of solutions is linearly independent, and (c), write the general solution. To do part (a), you should compute \( Mx_i \) and \( x_i' \) and show they are equal for each \( i \). Here \( M \) is the matrix in the given differential equation \( x' = Mx \).