DUE: Start of recitation, 2/10/15 (T3) or 2/12/15 (T2)

1.   a. Use Maple to find the roots of $y = x^3 - 3x^2 + 5x + 9$, to display these roots as $x1, x2$, and $x3$, and to graph the function on the interval $[-3, 3]$. You may use the sample code as a guide.

   b. Determine
$$\int_1^5 \frac{1}{\sqrt{3x^4 - x^2 - 2}}dx$$
   and briefly explain the result, using Maple Help and/or the Web.

2. Your sample code shows how to use Maple to find the general solution and particular solutions to the DE in problem 1.3.8 (excerpted below). Do the same for Problem 1.3.3 (also shown below), that is:

   a. Find the general solution to the DE of Problem 1.3.3.

   b. Find two particular solutions approximating the solution curves shown in Figure 1.3.17, by choosing appropriate initial conditions (ICs).
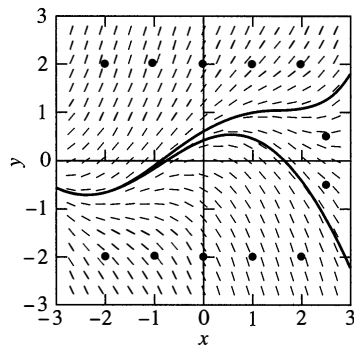
3. $\dfrac{dy}{dx} = y - \sin x$ **8.** $\dfrac{dy}{dx} = x^2 - y$
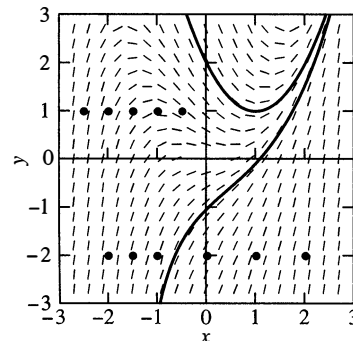

FIGURE 1.3.17.   **FIGURE 1.3.22.**

3. Your sample code shows how to generate the slope field and solution curves shown in Figure 1.3.22, for the DE in problem 1.3.8. Use this as a guide to generate Figure 1.3.17 (from Problem 1.3.3) using approximate ICs.

4. Consider the DE discussed in 1.4, Examples 2 and 3. Use Maple to generate a figure as similar as possible to Figure 1.4.2. Beware that Maple wont draw solution curves through singularities (where the explicit solution or the DE is not defined), so you will have to use more than three ICs to replicate the three solution curves, and your version of the solution curves will have gaps near the singularities. Make use of the general solution $H(x, y) = C$, where $H(x, y)$ is derived in Example 2.

Remark: Suppose, instead, that the initial condition in Example 1 had been $y(0) = -4$. Then it would follow that $y(x)$ is *negative* near $x = 0$. We should therefore replace $|y|$ with $-y$ in the integrated equation $\ln |y| = -3x^2 + C$ to obtain

$$\ln(-y) = -3x^2 + C.$$

The initial condition then yields $C = \ln 4$, so $\ln(-y) = -3x^2 + \ln 4$, and hence

$$y(x) = -4e^{-3x^2}.$$

This is the lower emphasized solution curve in Fig. 1.4.1. ∎

Example 2 Solve the differential equation

$$\frac{dy}{dx} = \frac{4 - 2x}{3y^2 - 5}. \tag{5}$$

Solution When we separate the variables and integrate both sides, we get

$$\int (3y^2 - 5)\, dy = \int (4 - 2x)\, dx;$$

$$y^3 - 5y = 4x - x^2 + C. \tag{6}$$

This equation is not readily solved for y as an explicit function of x. ∎

FIGURE 1.4.2. Slope field and solution curves for $y' = (4 - 2x)/(3y^2 - 5)$ in Example 2.

As Example 2 illustrates, it may or may not be possible or practical to solve Eq. (4) explicitly for y in terms of x. If not, then we call (4) an *implicit solution* of the differential equation in (2). Thus Eq. (6) gives an implicit solution of the differential equation in (5). Although it is not convenient to solve Eq. (6) explicitly in terms of x, we see that each solution curve $y = y(x)$ lies on a contour (or level) curve where the function

$$H(x, y) = x^2 - 4x + y^3 - 5y$$

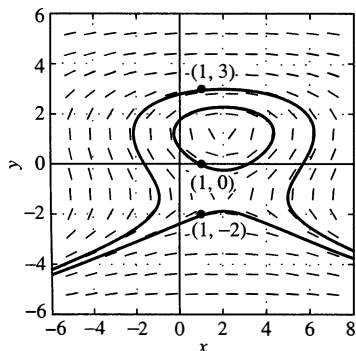is constant. Figure 1.4.2 shows several of these contour curves.

Example 3 To solve the initial value problem

$$\frac{dy}{dx} = \frac{4 - 2x}{3y^2 - 5}, \quad y(1) = 3, \tag{7}$$

we substitute $x = 1$ and $y = 3$ in Eq. (6) and get $C = 9$. Thus the desired particular solution $y(x)$ is defined implicitly by the equation

$$y^3 - 5y = 4x - x^2 + 9. \tag{8}$$

The corresponding solution curve $y = y(x)$ lies on the upper contour curve in Fig. 1.4.2—the one passing through $(1, 3)$. Because the graph of a differentiable solution cannot have a vertical tangent line anywhere, it appears from the figure that this particular solution is defined on the interval $(-1, 5)$ but not on the interval $(-3, 7)$. ∎

```
> x:= 'x';
```
$$x := x \qquad (1)$$

```
> solve(2*x^3-3*x^2+6*x+2 = 0);
```
$$-\frac{1}{2}\left(9+6\sqrt{3}\right)^{1/3}+\frac{3}{2\left(9+6\sqrt{3}\right)^{1/3}}+\frac{1}{2},\ \frac{1}{4}\left(9+6\sqrt{3}\right)^{1/3}-\frac{3}{4\left(9+6\sqrt{3}\right)^{1/3}} \qquad (2)$$
$$+\frac{1}{2}+\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(-\frac{1}{2}\left(9+6\sqrt{3}\right)^{1/3}-\frac{3}{2\left(9+6\sqrt{3}\right)^{1/3}}\right),\ \frac{1}{4}\left(9+6\sqrt{3}\right)^{1/3}$$
$$-\frac{3}{4\left(9+6\sqrt{3}\right)^{1/3}}+\frac{1}{2}-\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(-\frac{1}{2}\left(9+6\sqrt{3}\right)^{1/3}-\frac{3}{2\left(9+6\sqrt{3}\right)^{1/3}}\right)$$

```
> xsolns := solve(2*x^3-3*x^2+6*x+2 = 0,x);
```
$$xsolns := -\frac{1}{2}\left(9+6\sqrt{3}\right)^{1/3}+\frac{3}{2\left(9+6\sqrt{3}\right)^{1/3}}+\frac{1}{2},\ \frac{1}{4}\left(9+6\sqrt{3}\right)^{1/3} \qquad (3)$$
$$-\frac{3}{4\left(9+6\sqrt{3}\right)^{1/3}}+\frac{1}{2}+\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(-\frac{1}{2}\left(9+6\sqrt{3}\right)^{1/3}-\frac{3}{2\left(9+6\sqrt{3}\right)^{1/3}}\right),$$
$$\frac{1}{4}\left(9+6\sqrt{3}\right)^{1/3}-\frac{3}{4\left(9+6\sqrt{3}\right)^{1/3}}+\frac{1}{2}-\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(-\frac{1}{2}\left(9+6\sqrt{3}\right)^{1/3}\right.$$
$$\left.-\frac{3}{2\left(9+6\sqrt{3}\right)^{1/3}}\right)$$

```
> x1:= xsolns[1]; x2:= xsolns[2]; x3:= xsolns[3];
```
$$x1 := -\frac{1}{2}\left(9+6\sqrt{3}\right)^{1/3}+\frac{3}{2\left(9+6\sqrt{3}\right)^{1/3}}+\frac{1}{2}$$
$$x2 := \frac{1}{4}\left(9+6\sqrt{3}\right)^{1/3}-\frac{3}{4\left(9+6\sqrt{3}\right)^{1/3}}+\frac{1}{2}+\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(-\frac{1}{2}\left(9+6\sqrt{3}\right)^{1/3}\right.$$
$$\left.-\frac{3}{2\left(9+6\sqrt{3}\right)^{1/3}}\right)$$
$$x3 := \frac{1}{4}\left(9+6\sqrt{3}\right)^{1/3}-\frac{3}{4\left(9+6\sqrt{3}\right)^{1/3}}+\frac{1}{2}-\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(-\frac{1}{2}\left(9+6\sqrt{3}\right)^{1/3}\right. \qquad (4)$$
$$\left.-\frac{3}{2\left(9+6\sqrt{3}\right)^{1/3}}\right)$$

```
> plot(2*x^3-3*x^2+6*x+2, x = -3..3);
```
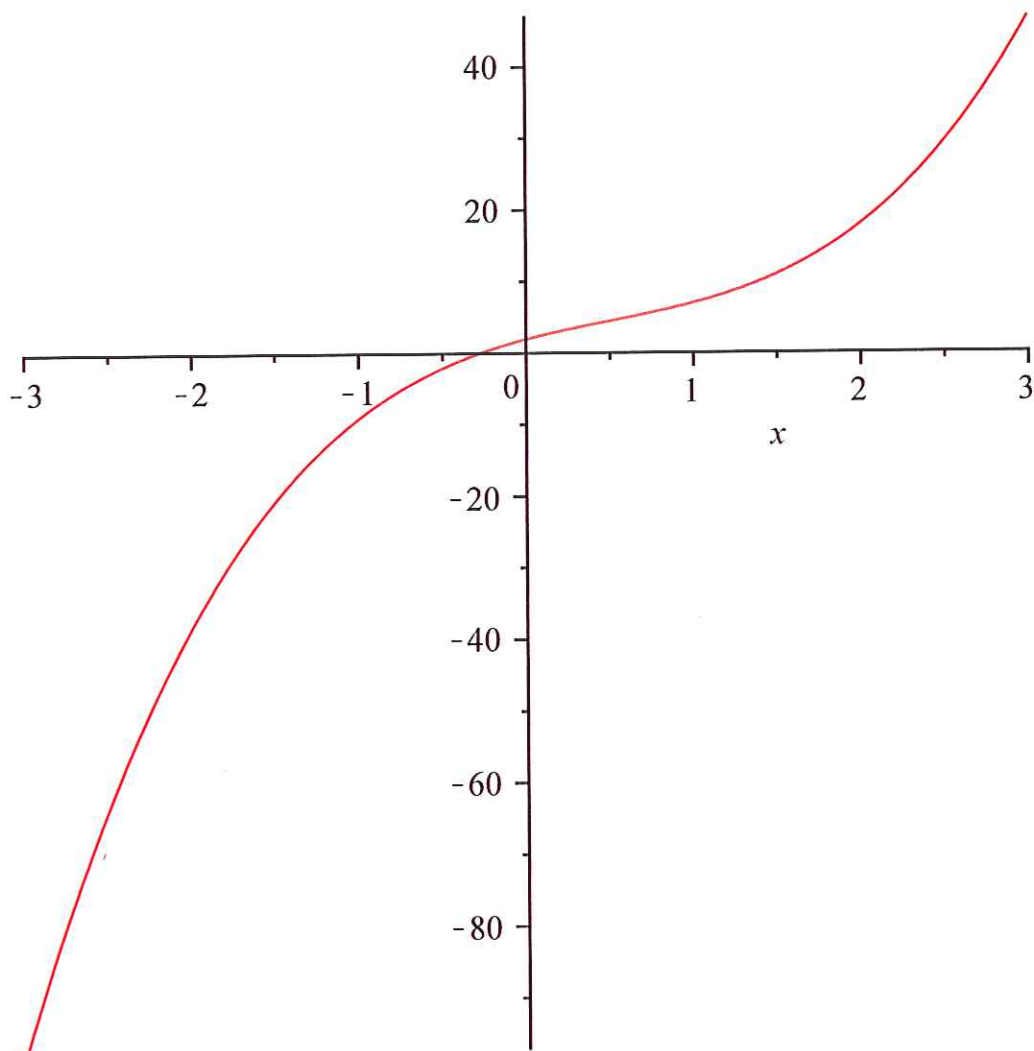
```
> ?int
```

$$\int_1^5 \frac{1}{\sqrt{3x^4 - x^2 - 2}}\,dx;$$

$$\frac{1}{5}\sqrt{5}\ \text{EllipticF}\left(\frac{2}{5}\sqrt{6}, \frac{1}{5}\sqrt{10}\right) \tag{5}$$

```
> with(DEtools):
> de:= diff( y(x), x) = x^2 - y(x);
```

$$de := \frac{d}{dx}\,y(x) = x^2 - y(x) \tag{6}$$

```
> yg:= dsolve( de, y(x) );
```

$$yg := y(x) = 2 - 2x + x^2 + e^{-x}\_C1 \tag{7}$$

```
> yp1:= dsolve( {de, y(0)= -1}, y(x) );
```

$$yp1 := y(x) = 2 - 2x + x^2 - 3\,e^{-x} \tag{8}$$

```
> yp2:= dsolve( {de, y(0)= 2}, y(x) );
```

$$yp2 := y(x) = 2 - 2x + x^2 \tag{9}$$

```
> DEplot( de, y(x), x = -3..3, y = -3..3, {[0,-1],[0,2]},dirgrid =
  [12,12], color = black, linecolor = blue, thickness = 2 );
```