Math 306 Section T

"Practice" Midterm Exam 1

February 17, 2015

Name: ______________________________________

This is a practice exam. For the real exam:

- Nothing on your desk except writing instruments and UB ID card.
- No electronics! I will keep track of time on the board.
- Like practice exam, the real exam has six questions, half of which involve qualitative analysis, curve sketching, and/or applications (note, you are not expected to memorize any specific mathematical models, just to be able to do the math in the context of a given model).
- Questions on the real exam are **not** guaranteed to be easier or harder than the practice exam; it's just for the sake of review and practice.
- Like the real exam, give yourself 80 minutes.

1. Find all solutions to the differential equation:

$$y^2y' + 2xy^3 = 6x$$

2. Find all solutions to the differential equation:

$$x(x + y)y' = y(x - y)$$

3. Consider the differential equation $\frac{dx}{dt} = (x+2)(x-2)x$

   (a) Find all equilibrium solutions and determine whether they are stable, unstable, or semistable.

   (b) Sketch a slope field for this differential equation.

   (c) Find the solution corresponding to the initial condition x(0) = 1.

   (d) Sketch the solution curve corresponding to your answer for (b).

4. Suppose a body moves through a resisting medium with resistance proportional to velocity, so that $\frac{dv}{dt} = -kv$. Let $x(t)$ be the position of the body, so that $v(t) = \frac{dx}{dt}$.

   (a) Solve for the function $x(t)$ in terms of the initial conditions $x_0 = x(0)$ and $v_0 = v(0)$.

   (b) Show that the moving body only travels a finite distance, by computing $\lim_{t\to\infty} x(t)$.

   (c) According to the model given by the differential equation $\frac{dv}{dt} = -kv$, if $v_0 > 0$, is velocity ever zero?

5. Consider the initial value problem

$$\frac{dy}{dx} = 2\sqrt{y} \qquad y(0) = y_0$$

(a) Find all solutions to the differential equation.

(b) For which y_0 does a unique solution exist?

(c) Show that, if $y_0 = 0$, two solutions exist. Explain why this does not contradict the theorem on existence and uniqueness of solutions to first-order ordinary differential equations.

6. Use the substitution $p = \frac{dy}{dx}$ to solve the differential equation below. You may leave your answer in terms of an integral if you use the fundamental theorem of calculus correctly.

$$2\sqrt{1-x^2}y'' - (y')^3 e^{-2x} = \sqrt{4-4x^2}y'$$