Math 306 Section T

"Practice" Midterm Exam 2

April 3, 2015

Name: _____

This is a practice exam. For the real exam:

- Nothing on your desk except writing instruments and UB ID card.
- No electronics! I will keep track of time on the board.
- Like practice exam, the real exam has six questions, half of which involve qualitative analysis, curve sketching, and/or applications (note, you are not expected to memorize any specific mathematical models, just to be able to do the math in the context of a given model).
- Questions on the real exam are **not** guaranteed to be easier or harder than the practice exam; it's just for the sake of review and practice.
- Like the real exam, give yourself 80 minutes.

1. Consider the differential equation:

$$y^{(3)} + 2y'' + 2y' = 0$$

   (a) How many linearly independent solutions do you expect, and why?

   (b) Find the general solution.

2. Solve the initial value problem:

$$y^{(4)} - 16y = 0$$
$$y(0) = 1 \quad y'(0) = 4 \quad y''(0) = 4 \quad y^{(3)}(0) = 0$$

3. Find the general solution to the differential equation:

$$y''(t) + 2y'(t) + y(t) = 2t\sin(t)$$

4. Find the general solution to the system of DEs:

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

5. Consider the system of DEs:

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 12 & -1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

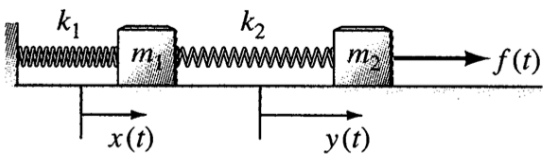
   (a) Find the general solution.

   (b) Find the solution satisfying ICs $x(0) = 1$ and $y(0) = 5$.

   (c) Sketch the phase portrait for this system.

6. Consider the mass-spring system on the left, derived from the diagram on the right:

$$x'' = -3x + 2(y - x),$$
$$y'' = -2(y - x)$$



Set $m_{1,2} = 1$, $k_1 = 3$, $k_2 = 2$, and $f(t) \equiv 0$.

   (a) Rewrite this system of two 2nd-order DEs system as a system of four 1st-order DEs.

   (b) Use the eigenvalue method to find at least two linearly independent solutions to your answer for (a).

   Note: Part (b) is shortened, in case you are attempting the practice exam within an 80 minute time limit, like the actual exam. Given more time, you should be able to find the full general solution.