Homework (week 1) — 537

Chapter 1: Mathematical review and computer arithmetic

- Your answers should be submitted to Naoki Masuda **by email (naokimas@buffalo.edu) by 12:30pm on Wednesday, September 4, 2019**.
- You should report not only the answers but how you reached your answers.
- Your codes, where they are required, should be included within the report (i.e. not to be submitted as separate files).
- Don't forget to write your name and ID.

1. Let $f' \in C[a, b]$ and $f'(x) \neq 0$ for all $x \in (a, b)$. Determine at how many points the function $f(x)$ can possibly vanish in $[a, b]$.

2. Write down a polynomial $p(x)$ such that $|\text{sinc}(x) - p(x)| \leq 10^{-5}$ for $-0.2 \leq x \leq 0.2$, where

$$\text{sinc}(x) = \begin{cases} \frac{\sin x}{x} & (x \neq 0), \\ 1 & (x = 0) \end{cases} \tag{1}$$

is the "sinc" function (well known in signal processing etc.). Prove that your polynomial $p(x)$ satisfies $|\text{sinc}(x) - p(x)| \leq 10^{-5}$ for $-0.2 \leq x \leq 0.2$.

Hint: You can obtain polynomial approximations with error terms for the sinc function by writing down Taylor polynomials and corresponding error terms for $\sin(x)$, then dividing them by x. This is easier than directly differentiating $\text{sinc}(x)$. For the proof part, you can use, for example, the Taylor polynomial remainder formula.

3. **[Skip this problem, which will be assigned in Week 2 homework]**. Note: To answer this question, you may want to code or use a calculator.

Suppose that we have a system with base $\beta = 10$, $t = 3$ decimal digits in the mantissa, and $L = -9$, $U = 9$ for the exponent. For example, 0.123×10^4, that is, 1230, is a machine number in this system. Also suppose that the "round to nearest" rule is used for rounding.

(a) What is HUGE for this system?

(b) What is TINY for this system?

(c) What is the machine epsilon ϵ_{m} for this system?

(d) Let $g(x) = \sin x + 1$. Write down $fl(g(0))$ and $fl(g(0.0008))$ in normalized format for this toy system.

(e) Compute $fl(fl(g(0.0008)) - fl(g(0)))$. Compare this to the nearest machine number to the exact value of $g(0.0008) - g(0)$?

(f) Compute $(fl(fl(g(0.0008)) - fl(g(0)))/fl(0.0008)$. Compare this to the nearest machine number to the exact value of $(g(0.0008) - g(0))/0.0008$ and to $g'(0)$.