

Homework (week 2) — 537

Chapter 1 and 2

- Your answers should be submitted to Naoki Masuda **by email (naokimas@buffalo.edu) by 1:00pm on Wednesday, September 11, 2019.**
- You should report not only the answers but how you reached your answers.
- Your codes, where they are required, should be included within the report (i.e. not to be submitted as separate files).
- Don't forget to write your name and ID.

1. Note: To answer this question, you may want to code or use a calculator.

Suppose that we have a system with base $\beta = 10$, $t = 3$ decimal digits in the mantissa, and $L = -9$, $U = 9$ for the exponent. For example, 0.123×10^4 , that is, 1230, is a machine number in this system. Also suppose that the “round to nearest” rule is used for rounding.

- (a) What is HUGE for this system?
 - (b) What is TINY for this system?
 - (c) What is the machine epsilon ϵ_m for this system?
 - (d) Let $g(x) = \sin x + 1$. Write down $fl(g(0))$ and $fl(g(0.0008))$ in normalized format for this toy system.
 - (e) Compute $fl(fl(g(0.0008)) - fl(g(0)))$. Compare this to the nearest machine number to the exact value of $g(0.0008) - g(0)$?
 - (f) Compute $(fl(fl(g(0.0008)) - fl(g(0)))/fl(0.0008))$. Compare this to the nearest machine number to the exact value of $(g(0.0008) - g(0))/0.0008$ and to $g'(0)$.
2. Consider the bisection method applied to $f(x) = \arctan(x)$, with initial interval $[a_0, b_0] = [-4.9, 5.1]$.
- (a) Are the hypotheses under which the bisection method converges valid? If so, then how many iterations would it take, mathematically, to obtain the solution to within an absolute error of 10^{-2} ? Answer this question without actually coding/running the bisection method.
 - (b) Code and run the bisection method (Algorithm 2.1). You can use the code available on the course website. Precisely, solve $f(x) = 0$ with $\epsilon = 10^{-2}, 10^{-4}, 10^{-8}, 10^{-16}, 10^{-32}$, and 10^{-64} . Report the solution and the number of steps needed, k , for each ϵ . If the method does not work (for some ϵ), explain why.
 - (c) Plot ϵ against k and explain the results. You should be interested in considering one or both axes to be shown on logscale. Think about the best way to show the results. Also be mindful to use legible (i.e. not too small) labels etc.