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# Study of simple land battles using agent-based modeling: Strategy and emergent phenomena

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In this paper, we expand upon our recent studies of an agent-based model of a battle between an intelligent army and an insurgent army to explore the role of modifying strategy according to the state of the battle (adaptive strategy) on battle outcomes. This model leads to surprising complexity and rich possibilities in battle outcomes, especially in battles between two well-matched sides. We contend that the use of adaptive strategies may be effective in winning battles.

Keywords: Cellular automata, battle, competition.

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## 1. Introduction

Study of social systems using statistical physics based approaches, or sociophysics, has been the subject of significant attention in recent years.<sup>1</sup> The theoretical works

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of Galam and collaborators on political events such as elections including the 2016 election of Trump<sup>3,a</sup> have illustrated the predictive power of sociophysical studies. Studies of gang rivalries using statistical physics based approaches have also emerged in recent years.<sup>2</sup> Cellular automata<sup>4</sup> or agent-based models<sup>5</sup> and even molecular dynamics based studies<sup>6</sup> have likewise proven to be a powerful tool to model the temporal evolution of various complex systems.<sup>7,8</sup> While technically simple to set up, these models often can reveal deep insights into the dynamics of complex systems such as those alluded to below, behaviors that may not be discernible using traditional differential equation based models.<sup>7</sup>

Cellular automata have naturally found applications in modeling such diverse phenomena as crowd behavior,<sup>9,10</sup> emergent behavior of an organization under specific conditions,<sup>11</sup> market behavior,<sup>12</sup> in capturing the success of competing brands or products in the market,<sup>13</sup> in successfully describing the spread and mitigation of epidemics such as bird flu in poultry,<sup>14,15</sup> and in describing the struggle between animal species for territory and resources.<sup>16</sup> Here, we use an agent-based approach to examine simple land battles, and explore various effects of initial conditions and strategy on the evolution and the outcome of such battles.<sup>17,18</sup> Our battle model currently describes a conflict between two species, though it is possible to extend this to multiple species (see e.g., Ref. 19).

We construct a simple physical model consisting of a two-dimensional square lattice or matrix and simple deployment rules.<sup>17,18</sup> The symbols we use in this study are introduced in Table 1 so that the work is easier to follow. The lattice represents the battlefield on which conflict takes place. Individual attackers and defenders are assigned numeric strength values and initially placed in the reserves. The simulations involve deployment strategies which result in occupation of the array sites by the combatants. Casualties result when opposing forces occupy the same site. Combatants do not actually move on the battlefield, but rather occupy their given sites, engage intruders, and provide field information which may be used for determining the positioning of troops or agents in future iterations. Battles continue until either the attackers or the defenders have exhausted their reserve forces, or alternatively until either the attackers or defenders have gained control of the entire battlefield. If both sides have essentially deployed all forces to the conflict, the battle is a draw. While this win/loss definition is somewhat arbitrary, it allows for a general discussion of results. To measure a victory numerically, we use a so-called win/loss ratio  $\chi$  as defined in Eq. (3).

### 2. Model Details

Let us begin by considering a simple battle on a square lattice of sides L = 5. At the start of the battle, each side is allotted an initial reserve of  $A_0 = D_0 =$ 

<sup>&</sup>lt;sup>a</sup>It is interesting to see how Galam correctly projected the strong possibility of Trump becoming the President-elect of the US nearly ten months in advance when the US polls largely failed to make the correct projection. See S. Galam, in this issue.

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L	is the length of each side of the square lattice
$A_0, A$	initial/final attack reserves
$D_0, D$	initial/final defense reserves
$ ho_A,  ho_D$	attack/defense density
a, d	attack (negative), defense (positive) force level
R	sight range (number of cells)
r	sight range (percentage of battlefield size)
$\chi$	win–loss ratio

Table 1. Description of notation used.

5000 which may represent weapons, resources, troops, or any other logistical factor (see Table 1). A and D, respectively, denote the numbers of "attackers" and the "defenders" at any given time other than at the initiation point; we distinguish them by means of the strategy each uses. To initialize the battle, we randomly place the defender's force on the battlefield. This is done by taking a fixed number of agents from their initial reserve  $D_0$  and placing them randomly upon the lattice (with a given density  $\rho_D$  which is the ratio of sites occupied by the defenders to the total number of sites on the lattice). Assuming that the defenders have various forms of weapons and ammunitions, the deployment is done such that there is a mix of strength values of, say, d = +1 and d = +2.<sup>b</sup> After this step, each lattice site K(i, j) = 0, 1, or 2.

The battle then begins with the attackers deploying in random locations on the battlefield with their own occupation density  $\rho_A$ , with strength values of a = -1 and a = -2. We pause to note that this is what we will refer to as a "symmetric" battle, since the strength value distributions of the attackers and defenders are equal in magnitude.

Following this initial deployment, some fraction of the sites will now have negative values, indicating they are now held by the attackers. On subsequent iterations, the attackers and defenders will continue to draw from their respective reserves according to different algorithms. One can envision that such reserves are pulled from distributed command/control centers where reserve forces await.

Figure 1 shows an example randomized lattice at the start of a battle on a  $5 \times 5$  lattice. For our first example, we will use a random strategy for defenders and an intelligent, aggressive strategy for the attackers. This is inspired by the recent conflicts in Iraq; we choose to study the case of an insurgent army with access across the entire battlefield and formidable knowledge of the terrain (and therefore highly capable of making unpredictable or random attacks), versus an "intelligent" army whose decision-making is limited by information regarding the distributions of the attackers and the defenders.

<sup>&</sup>lt;sup>b</sup>It is interesting to observe that the values of d and a are roughly connected to nature of arms available in the battle. Thus, d = +1 or a = -1, etc. would mean relatively simple weapons whereas d = +2, etc. would correspond to the availability of weapons that can cause more casualties.

1			76				2		16
2	-1	0	-2	1	173	-31	190	-115	199
-1	-1	-1	1	-1	-347	97	7	113	-31
1	-1	1	1	2	184	-101	194	-409	191
-1	2	-1	-2	-1	-233	162	126	170	-1
0	0	0	-1	-1	146	-308	211	- <mark>1</mark> 92	190
(a)							(b)		

Fig. 1. (a) The initial randomized state of a battle on a square lattice of sides L = 5. (b) The final state, after reserves have been depleted for the defenders.

The defenders will continue to deploy their reserves as in the first step throughout the battle, by randomly selecting lattice sites with probability  $\rho_D$  and placing randomly d = +1 or d = +2 on that site on each iteration.

For the attackers, it is important to know where the greatest density of defenders are located on the lattice. This knowledge requires intelligence regarding the current state of the battlefield and this intelligence turns out to be important in determining whether the attackers can overpower their opponents. In our studies discussed below, the attackers take an aggressive approach by always deploying resources in the direction with the greatest density of defenders though, as we shall see later, the reverse (defensive) strategy of retreating from the regions of greatest density of defenders can also turn out to be an effective strategy in some cases. We define a neighbor list by an  $n \times m$  matrix about each attack site,  $K_A(i, j)$ . The size of the neighbor list is dictated by the range variable R, the number of lattice cells in any direction for which information is available at site  $K_A$ . For an  $L \times L$  matrix, we may also measure the range as a fraction of lattice size:  $|r| = \frac{R}{L}(100)\%$ . The attackers receive information from a neighbor list with maximum size  $(2R+1) \times (2R+1)$ . The occupation or resource values of all sites in each quadrant of the neighbor list at  $K_A(i, j)$  are summed. If the computed sum is largest in, say, the first quadrant, then site in which the attackers will be deployed to is

$$K(i+1, j+1) \to K(i+1, j+1) + a_{\max},$$
 (1)

(if this site is still within the battlefield) and so on. If adjacent quadrants, for example quadrants I and II, have equal values

$$K(i, j+1) \to K(i, j+1) + a_{\max}$$
. (2)

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Observe here that a larger value of R implies averaged and hence less precise information over a larger area of a quadrant. Further, if R is very small, that too would imply limited intelligence about the immediate battlefield environment. Hence very small Rs are typically not very meaningful for our purposes. Simply put, larger R implies limited battlefield intelligence whereas small R would typically imply strong local battlefield intelligence.

If opposing quadrants (i.e., I and III, or II and IV) or if three or even all four quadrants of the neighbor list have the same value, there is no new deployment from that site during that particular iteration (however, this case seldom occurs). The rationale for using maximum strength values for the attackers is their strategic need to maximize effectiveness. The battlefield is not updated until all such decisions have been made.

At each iteration, these deployments repeat until at least one of the initial reserves  $A_0$  or  $D_0$  has been completely depleted; at this point the battle ends, and we must decide who has won!

For convenience, we will measure the magnitude of a victory by means of the single number  $\chi$ , the win–loss ratio, defined as follows:

$$\chi = \begin{cases}
-1, & \rho_D = 0, \\
+1, & \rho_A = 0, \\
\frac{D-A}{D_0 + A_0} & \text{otherwise.} 
\end{cases}$$
(3)

This definition serves to measure victory of two types: by total territory control, or by depletion of reserves. Furthermore, the more lopsided the victory toward one side, the larger a value  $\chi$  will take. Figure 1 shows the initial and final lattice distributions of a simple battle of the type described above.

For the rest of this paper, all battles will take place upon a  $50 \times 50$  lattice unless otherwise noted; on a lattice as small as the one above, the intelligent strategy of the attackers is nearly irrelevant due to the neighbor list covering most of the lattice! We have also studied lattice sizes that are large as L = 2500 and found that dealing with such large systems increases the computation times needed but has no measurable effect on the battle outcomes.

### 3. Strategy

We use the term "strategy" very broadly; it represents the force composition and the behavior of one side, as well as their set of reactions to enemy moves. Our overall objective with strategy adjustment is to find a way to optimize one side's chances of winning; that is, the best ways to use intelligence to win a symmetric battle.

### 3.1. Initial conditions

Let us begin by studying the simple battle described in the previous section with aggressive (as opposed to defensive), intelligent attackers, but placed in a larger lattice. Here we wish to study the initial conditions that lead to a high probability for the attackers to win. We note that the simulation *is* susceptible to different victory/loss outcomes from different seeds of the random number generator; for this reason, whenever a battle with certain initial conditions is mentioned, we are describing an *average* result taken over seven battles with different seeds.

To do this numerically, we examine the decay of the attackers' reserves (denoted by  $A_0$  for initial and by A for instantaneous values including the final value when the simulation ends) as a function of the parameters (i) sight range, (ii) initial attack density, and (iii) initial defense density. The battles we studied were with symmetric force levels a, d = -1, +1; -2, +2; and -3, +3; and asymmetric battles with force levels a, d = -2, +5; -3, +5; and -4, +5 (e.g., for -2, +5, a = -2, -1 and d = +1,+2, +3, +4, +5 are possible deployment values). The symmetric battles represent two armies with matched weapons, equipment, and reserves; the asymmetric battles give one side an advantage in terms of effectiveness of the attacks.

After some trial and error, we ended up varying the following parameters (see Table 1) through the following ranges:

$$0.2 \le \rho_A \le 0.85$$
,  
 $0.2 \le \rho_D \le 0.85$ ,  
 $1 \le R \le 20$ ,

where  $\rho_D$  is held fixed at all times whereas  $\rho_A$  is fixed at t = 0 and allowed to evolve based on the evolution of the battle. Henceforth, unless otherwise specified, by  $\rho_A$ we mean  $\rho_A(t=0)$ .

These limits on the densities  $\rho_A$  and  $\rho_D$  of attackers or defenders on the lattice exclude uninteresting combinations of  $\rho_A$ ,  $\rho_D$  and R. For instance, places where the attackers deploy so few troops that they lose the battle within one or two iterations. Since R is the sight range in number of cells, it must be an integer (for perspective, 20 cells is 40% of the L = 50 lattice battlefield.)

Let us assume that the attackers take a small fraction of agents from the initial reserves for every deployment and keep repeating this process until the reserves are depleted. Let us further assume that A would depend on the range R with A depleting more rapidly as R grows. Pretending for now that R is a continuous variable (and  $R \to 0$  is feasible) it would be reasonable to assume  $dA/dR = -\alpha A$ , where  $\alpha$  is some constant that depends on  $\rho_A$  and  $\rho_D$ . Then one can write

$$\int_{A_0}^{A} \frac{dA'}{A'} = -\alpha \int_0^R dR'$$
or  $A = A_0 \exp(-\alpha R)$ ,
(4)

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Fig. 2.  $\log(A)$  versus R, with several different values for the initial value of  $\rho_A$ .

where the primes in Eq. (4) denote dummy variables. The parameter  $\alpha$ , then, tells us how strongly the outcome of the battle is coupled to the sight range parameter for the chosen attack density. Equation (4) suggests that when the sight range is too high, the attackers suffer very high losses. Figure 2 presents the results from our simulations which show that the exponential decay above provides a reasonable description of how A decays as a function of R. Observe that  $\alpha$  would have a higher magnitude if  $\rho_A$  is smaller.

Let us look closely at the decay parameter  $\alpha(\rho_A, \rho_D)$  which controls how the reserves of the attackers get depleted. As  $\rho_D$  becomes large and  $\rho_A$  becomes small,  $A \approx A_0$ , which means that attackers cannot deploy any troops. This occurs because the field is so concentrated with defenders that the attackers cannot find a place to deploy. In the opposite limit,  $\rho_D$  is small and  $\rho_A$  is large;  $A \approx 0$ . This is caused by a lack of defenders in the field, which will make attacker's territory occupancy quickly grow — leading them to either win by capturing all territory, or lose by running out of reserves too quickly. So, for fixed initial conditions, we can summarize what the outcome of the battle will be in terms of the single parameter  $\alpha$ .

Our simulations show that  $\alpha$  itself undergoes an exponential decay as a function of  $\frac{\rho_D}{\rho_k^k}$  as (see Fig. 3)

$$\ln|\alpha| = -\gamma \frac{\rho_D}{\rho_A{}^k} + \delta \,, \tag{5}$$

$$|\alpha| = \exp(\delta) \exp\left(-\gamma \frac{\rho_D}{\rho_A^k}\right).$$
(6)

 $\gamma$  and  $\delta$  are positive constants for a given symmetric force level, and k is approximately 0.20, for all cases analyzed.



Fig. 3.  $\alpha$ , the decay parameter of Fig. 2, as a function of defense density  $\rho_D$ .

Combining Eqs. (4) and (6),

$$A = A_0 \exp\left(-Cr \exp\left(-\gamma \frac{\rho_D}{\rho_A{}^k}\right)\right), \qquad (7)$$

where  $C \equiv \exp(\delta)$ . Equation (7) summarizes how the attackers' reserves will behave as r,  $\rho_D$ , and  $\rho_A$  change.

We find therefore that the outcome of the battle for the aggressive attackers depends greatly on *moderation*: one must have *some* local knowledge, but not too much! One must deploy *enough* troops from reserves, but not too many! We next briefly summarize our studies on the effectiveness of the defensive strategy [see discussion above Eq. (1)] before moving on to the strengths and weaknesses of various mixed strategies.

Figure 4 summarizes the results from a large number of studies determining how much territory has been acquired by the attackers for varying r given different  $\rho_A$ ,  $\rho_D$  and strategies, i.e., whether aggressive or defensive. In Figs. 4(a)–4(c), |a| = |b| = 1, and in Figs. 4(d)–4(f), |a| = |b| = 2. Each point represents the average of three runs with associated error bars. In each panel, the percentage of sites held by the attackers is shown versus r for fixed values of  $\rho_A(0)$  and  $\rho_D$ . In panels (a) and (d) we have used  $\rho_A = 0.2$ ,  $\rho_D = 0.45$ , in (b) and (e)  $\rho_A = 0.40$ ,  $\rho_D = 0.40$ , and in (c) and (f)  $\rho_A = 0.45$ ,  $\rho_D = 0.20$ . We summarize the results as follows:

When |a| = |b| = 2, the conflict outcomes become more sensitive to the value of r and to whether an aggressive or defensive strategy is used. Our studies suggest that when the defenders are highly exposed to attacks, but not so high as to exhaust the reserves too quickly, such as in Figs. 4(c) and 4(f), it is less likely that the attackers

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Fig. 4. Lattice site occupation percentage of attackers versus range for aggressive and defensive strategy battles are shown for symmetric force levels in (a)–(c) for |a|, |b| = 1, and in (d)–(f) for |a| = |b| = 2. The magnitudes of  $\rho_A$  and  $\rho_D$  are given in Table 2. The symbols used are as follows: *Black diamond, filled*: indicates the defenders have eradicated the attackers from the lattice and simulation ends. *Black diamond, open*: indicates the attackers have eradicated the defenders from the lattice. *Open stars*: denote that the defenders have run out of resources first; hence the attackers have won. From the above discussions, more attacker wins are expected for low  $\rho_A = \rho_A(0)$  and high  $\rho_D$ . The open stars in our results are consistent with expectations. *Filled stars*: denote situations where the attackers have run out of resources first, resulting in a defender victory. *Grey circles, filled*: are used to show cases where the attackers and the defenders both exhaust resources at essentially the same time, with the contest then resulting in a draw.

Table 2. Details of parameters used in the studies in Fig. 4.

Figure 4	$ ho_D$	$ ho_A$	Force levels
(a)	0.45	0.20	$\pm 1$
(b)	0.40	0.40	$\pm 1$
(c)	0.20	0.45	$\pm 1$
(d)	0.45	0.20	$\pm 1, \pm 2$
(e)	0.40	0.40	$\pm 1, \pm 2$
(f)	0.20	0.45	$\pm 1, \pm 2$

will win. We also find that in nearly all of our analyses shown in Fig. 4, that the defensive approach results in less area of the lattice controlled by the attackers but provides a better chance of dominating the defenders. The only exception we found

is when the attackers have low exposure and high local intelligence and the force levels are minimal, a defensive strategy works better in winning *and* in establishing territorial dominance in favor of attackers. We note from the error bars that the effects of randomness on the spatial and force outcomes increase with increasing  $\rho_A$  and with increasing attack strength.

From here, we will seek to further optimize these battles by adjusting the attackers' strategy — for example, is it possible to reduce that minimum deployment density  $\rho_A$  in order to reduce losses? This and more will be discussed in the following section.

#### 3.2. Mid-battle strategies

In a basic battle, the attackers' strategy is very simple. Both sides initially deploy randomly; the defenders continue to do so for the duration, but the attackers deploy reinforcements in the direction of high defender concentrations. This is what we had called an aggressive strategy. A defensive strategy is the opposite; attackers deploy in the opposite direction of enemy concentrations (i.e., they always retreat).

The aggressive strategy tends to be excellent for capturing *territory*, but takes heavy losses to do so. The defensive strategy has the opposite effect; it cannot capture territory well, but it minimizes losses. Our initial attempt using a mix of strategies was simply to vary the aggressiveness of the attackers during the course of the simulation. For example, have the attackers flip a coin at each iteration to decide whether they will use an aggressive or defensive strategy. However, after some testing, we found that this is an ineffective strategy; it shows the weaknesses of both of the aggressive and defensive strategies, but does not adequately capture their strengths. The studies hence suggested that actions must be based on sound logic aimed at neutralizing the enemy rather than taking chances with what could be ineffective moves.

To improve the adaptive strategy approach, we made it more active. We designed the strategy to hold site occupation between 20 and 80% of the battlefield; from previous work, this is a favorable region of the phase space for the attackers (see Figs. 9–11 in Ref. 20). The mechanics are simple: when site occupation is low, we increase the attackers' force level and aggressiveness, as if they had brought heavier weapons from a reserve. When site occupation is too high (i.e., they become overextended), we switch the attackers' strategy to fully defensive. We have observed that this causes them to retreat into small  $(2 \times 2 \text{ or } 3 \times 3)$  regions of high troop density — fortified positions, in effect.

Unlike the much simpler mixed strategy, the adaptive strategy is very powerful; it results in attacker victories in many regions of the phase diagram where they would normally have lost. See Fig. 5 for an example.

This strategy still is not quite optimized, however; at a defense density near 40%, the attackers still nearly always lose despite their cleverness. We use a genetic algorithm to quickly search through the space to find a winning strategy — in



Fig. 5. (Color online) Example phase diagram for the adaptive strategy case. Here,  $\rho_D$  is set to 60%, and the cutoff points for switching to aggressive or defensive strategies are set at  $\rho_A = 20\%$  and  $\rho_A = 80\%$  of the lattice, respectively. On the left is a fully aggressive strategy, and on the right is the adaptive strategy. Red and blue indicate regions where the attackers and defenders win, respectively; the color scale represents  $100 \cdot \chi$ .

particular, we try to minimize attacker casualties while maximizing defender losses.<sup>21</sup>

As stated above, adaptive strategy involves force level and aggressiveness modifications. We allow the genetic algorithm to control the following parameters: the site occupation thresholds determining when the new strategies will be applied, and the magnitude of the changes. The fitness function is determined by running a battle using these details (see e.g., Ref. 21), and calculating  $\chi$ .

Most battles end by having one side or the other run out of reinforcements; therefore, our victory condition is scaled based on how many troops remain for each side. A genetic algorithm, of course, tends to exploit weaknesses in maximizing its fitness function, and this is precisely what happens here. However, we hoped to find some intuition in the way it did so. The algorithm causes the attackers to adopt a totally defensive strategy for site occupation greater than about 3–5%. Under that, it significantly increases their force level and makes them fully aggressive. This allows them to keep one or two small "strongholds" during the entire battle and deplete the defense forces, who are forced to deploy over the entire battlefield. This ends in an extreme victory for the attackers; it is not uncommon for them to have hundreds of thousands of reserves against zero defender reserves. We have not shown a figure for the phase diagram here, simply because it is not very interesting — it is *entirely* blue, i.e., the attackers have won every battle when this optimized strategy is applied. Of course, it is important to remember that this strategy works best against the random deployment of the defenders in this model. Studies on the effectiveness of the adaptive strategy against strategic defenders will be pursued in the future. Some preliminary work along these lines is summarized below.

To make the defense more effective while preserving its randomness, we used the idea of a cell-structured defense. In this strategy, the defenders are positioned only near a few particular points on the battlefield. This results in small, tough concentrations of troops scattered about, much like the "strongholds" seen when

the attackers use an adaptive strategy. The difference here is that a cell may be defeated if its leader is killed; that is, if the attackers manage to penetrate to the center of the cell, defenders stop deploying there. Against any enemy short of the genetic algorithm based optimization strategy, this leads to a victory for the defense.

To adjust for the cell defense, we used a strategy called "smart deployment" for the attackers. This strategy assumes that the attackers have excellent intelligence of the battlefield, and causes them to deploy only in regions where there is a heavy enemy presence. It completely countered the cell structure, returning our battles to a contested state. The reason for this result is twofold: first, it was applied to an army that has local intelligence. This allows the attackers to quickly locate and deploy near the main concentrations. Second, the battlefield is far emptier than in a basic battle; since the defenders only control the immediate area around their cells, the attackers can gain the benefit of not deploying in the empty areas.

The battle with cell-structured defense and smart-deploying attackers has many interesting behaviors. The attackers surround defense concentrations, cut through lines, and push forward to the center of the cells. Figure 6 shows a snapshot taken from one of these battles.



Fig. 6. (Color online) An intelligent attacker using smart deployment against a cell-based defense. Black represents defenders, gray represents attackers, and white cells are neutral.

### 4. Conclusion

We have discussed a simple two-dimensional battle model with intelligent armies using various strategies. Strategies which are both highly aggressive and highly conservative must be used in careful balance to gain victory. Furthermore, we investigated the cases of insurgent defenders using both randomized and cell-based deployments, which are far more successful. We have been able to see trends in our calculated states of the battles and we have been able to predict the outcomes of battles based on their initial conditions. We hope that this model can help find useful strategies and predict outcomes in real life battles and in related problems involving competitions between two or more parties in other contexts such as business, species survival and others.

At a broader level, our studies show that an organized and goal-oriented operation can almost always overcome a disorganized resistance, even when the strengths are comparable. Perhaps the study ultimately points to the adage concerning survival of the fittest where fitness is measured in intelligence and hence in tactical terms.

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