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ORIGINAL PAPER

Decorated granular layers for impact decimation

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Abstract We present dynamical simulations and simple mechanics arguments to propose a system of stacked blocks of square lattices of elastic spheres that can be used to decimate an incident impulse. Mass mismatch between adjacent blocks is accomplished by making the sphere radius in the upper block twice that of the lower block. The system decimates impact energies by converting the initial impulse into two solitary waves and then progressively into many smaller amplitude solitary waves. We also show that near perfect impact decimation capability can be realized with increased mass mismatch between adjacent blocks by creating sandwiched structures in which a block with smaller density spheres is surrounded on both sides with blocks of denser spheres. The proposed systems are expected to be scalable down to spheres of ∼100 nm and work for solid and hollow spheres.

Keywords Square granular lattice · Impact absorbing metamaterials · DEM

1 Introduction

An impulse propagates as a non-dispersive energy bundle or a solitary wave through an alignment of monosized spherical grains [1–7], whether these grains are solid or even hollow [8]. These waves are a result of the strongly nonlinear nature of the interaction between the grains [9], and exhibit unique properties such as nonlinear dependence of the wave speed on the force, and the width of the wave being independent of its amplitude. Dissipative losses exponentially attenuate the solitary wave in space and time but often such attenuation sets in slowly [10,11]. These properties of the system are scalable in the sense that they do not depend on the size of the grains as long as they behave as elastic objects. Studies suggest that grains behave as elastic objects at diameters of ∼100 nm or even less [12,13]. Furthermore, the flexibility in designing grains with different materials and geometry and thereby tuning the solitary wave response has led to many engineering applications such as in shock absorption and acoustic focusing devices [see for e.g. Ref. [14] and references therein]. It is now well understood that introducing a mass mismatch between the interface of two grains such as in $1D$ tapered and decorated chains [15–21] by changing the size of the grains or by designing composite chains [22–25] with grains of different materials results in disintegration of the impulse and therefore these systems serve as effective $1D$ system for impulse decimation. Impact decimation is a topic of enormous importance in insuring protection of structures and equipment. In different energy scales, impact decimation is needed in protecting structures from large storms, earthquakes and related hazards. On a smaller scale, impact decimation is important in the context of protecting delicate electronics, in protecting people from traumatizing head injuries and similar applications. The overall effectiveness of a particular arrangement would depend both on the specific application and the ability to organize them in higher dimensions.

Impact absorption in $3D$ using granular systems is a subject of intense examination [26–28]. The study of impact

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propagation in $2D$ granular systems was pioneered by Shukla et al. [29,30] and later examined by others. Recently Leonard et al. [31] have explored impact decimation in network structures, a closely related problem. Here we propose a purely $2D$ system made of stacked arrays of square lattices of spherical grains[1]. The first block where the impact is incident is made of the largest spheres in the system. The spheres progressively shrink in radius by a factor of 2 as one goes into arrays or blocks deeper down. The system exploits the presence of solitary waves and of strategically dividing the energy in the solitary waves through increasingly many chains of progressively smaller radii. We show below via dynamical simulations and simple phenomenological arguments how such a system works and suggest that $2D$ systems or layers can be stacked to design blocks which can rapidly absorb impulses incident along the c-axis.

2 Model

The proposed system with two stacked blocks is shown in Fig. 1a. At the interface between the two blocks the larger sphere of radius $2r$ is placed symmetrically on top of the two smaller spheres of half its radius in the block below. The center of the larger sphere is at a distance of $2\sqrt{2}r$ from the midpoint of the line joining the centers of the smaller spheres. This ensures a stable packing and also results in a nested arrangement, since, each vertical column of spheres in a block is supported by two columns in the block below. Multiple stacked block panels can be used to design the impact decimation container. As shown in Fig. 1b we can think of many stacked panels arranged in parallel with a step-shaped insulating matrix filling the space between two panels. The width of the insulating matrix can go to zero for the top block and increases by the radius of the sphere for consecutive blocks.

Dynamical simulations on the stacked block arrangement in Fig. 1a are performed using the discrete element method (DEM) based parallel open-source DEM software LIGHHHTS (LAMMPS improved for general granular and granular heat transfer simulations) [32–34]. DEM is based on contact mechanics, where, for two particles in contact the total force at the point of contact is decomposed into a normal part F_n and a tangential part F_t, which in the absence of any dissipation are given by,

$$F_n = k_n \delta_n, \tag{1}$$

$$F_t = k_t \delta_t, \tag{2}$$

δ_n and δ_t are the normal and tangential overlap respectively. For two spheres with separation r and radii R_1 and R_2 the normal overlap $\delta_n = (R_1 + R_2) - r$ and the contact force is nonzero only when $\delta_n > 0$. The tangential overlap δ_t is the relative tangential displacement between two spheres for the entire contact duration and the tangential force is limited by the coulomb criteria $F_t = min(F_t, \mu F_n)$. We use the nonlinear Hertz contact interaction for describing the normal force in our simulations [35]. The spring constants k_n and k_t therefore depend on the normal overlap in addition to the material parameters, $k_n = 4/3Y^*\sqrt{R^*\delta_n}$ and $k_t = 8G^*\sqrt{R^*\delta_n}$. For spheres with the same Young's modulus Y and Poisson ratio ν the effective Young's modulus $Y^* = \frac{Y}{2(1-\nu^2)}$ and the effective shear modulus $G^* = \frac{Y}{4(2-\nu)(1+\nu)}$. R^* is the effective radius and equals $\frac{R_1 R_2}{R_1+R_2}$. The time step of simulation is taken to be 10^{-9} s. For simplicity, the striker is assumed to be of the same size and material properties as the spheres in the top block and the impact point is taken to be inbetween the two top spheres as shown in Fig. 1. In our simulations we observe that the sideways propagation of energy is reduced significantly in the presence of static friction. For μ values of 0.1 and 0.25 the energy flowing sideways in the uppermost layer of the top block reduces by almost 44 and 75 % respectively. The energy propagating vertically downwards, however, does not show any significant variation. We therefore focus only on the downward propagating component of the impact energy.

3 Results and discussions

To outline the central idea behind the impact decimation capability of the proposed system, let us first consider a system made from blocks large enough to allow for the formation of fully developed solitary waves. The kinetic energy in different blocks of a four block system of steel spheres with density $\rho = 8000$ kg/m^3, Young's modulus $Y = 193$ GPa and Poisson's ratio $\nu = 0.3$ is shown in Fig. 2. The radius of the spheres in the lowest block is taken to be 1 mm. As can be seen in Fig. 2 a single solitary wave generated upon impact by the striker in the top block splits into a series of smaller amplitude solitary waves after crossing the interface to the next block. Presence of more blocks results in further splitting and amplitude reduction at each of the subsequent interfaces. This behavior is typical of solitary wave train generation in 1D systems when the solitary wave passes from denser to rarer medium [36,37]. In the proposed system, following a hard sphere model, it is simple to show through momentum and energy conservation that the ratio of the final speed v_f to the initial speed v_i of the top sphere at the interface is given by

$$\frac{v_f}{v_i} = \left(\frac{1 - 2\kappa\cos^2\theta}{1 + 2\kappa\cos^2\theta}\right), \tag{3}$$

[1] This idea grew out of a suggestion along similar lines by Professor F. Melo in 2009.

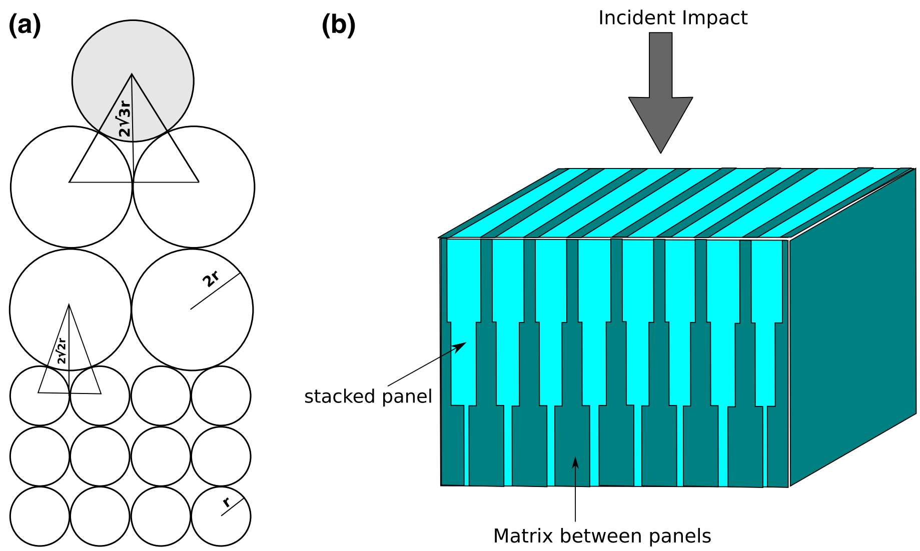

Fig. 1 (*Color online*) **a** Picture of the 1-dimensional granular strip. **b** Schematic of the proposed impact decimation container with many stacked panels arranged in *parallel* and an insulating matrix sandwiched between two panels

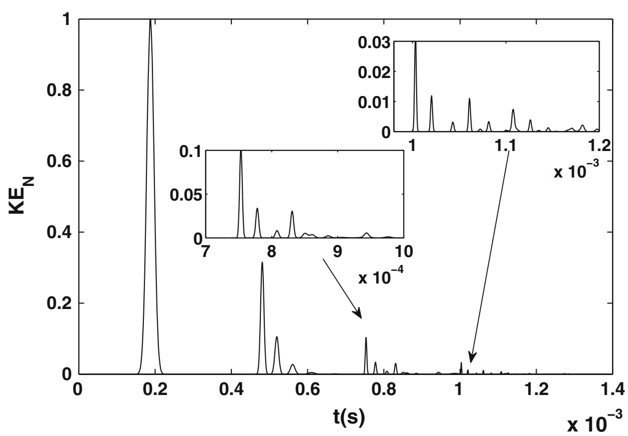

Fig. 2 Normalized kinetic energy away from the interface in a four block system. The *inset* shows the kinetic energy in the third and fourth block respectively

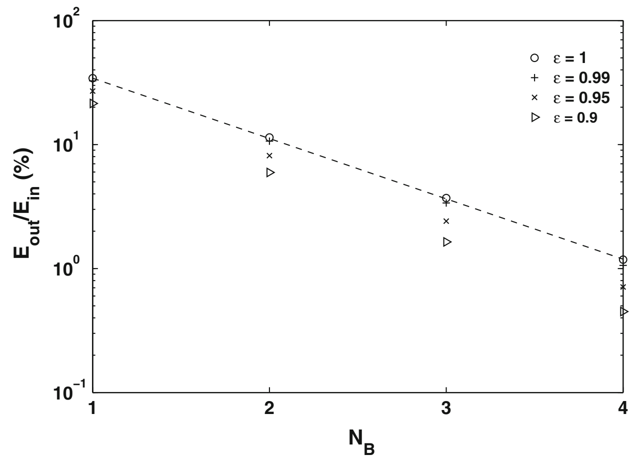

Fig. 3 Semilog plot of the fraction of output kinetic energy for systems with different numbers of blocks N_B and restitution values. The total width of the setup is $\sum_{i=1}^{N_B} 2^{3+i} r$. The radius r of the spheres in the lowest block is 1 mm

$\kappa = m_2/m_1 = (\rho_2/\rho_1)(r_2/r_1)^3$ is the ratio of the mass of the smaller sphere to the larger sphere and equals 1/8 for spheres of the same density ρ and $r_1 = 2r_2$, $\cos\theta = 2\sqrt{2}/3$ (see Fig. 1a). Since the final velocity is positive, the larger sphere continues to move in its original direction after impact, and, multiple interactions with the lower sphere leads to the generation of solitary wave trains [38]. Apart from the splitting which is a consequence of mass difference, energy also gets distributed in twice the number of columns in consecutive blocks due to the nested arrangement of the spheres at the interface. For the four block system considered here the kinetic energy gets distributed in two, four, eight and sixteen columns in subsequent blocks. The shock decimation capability of this system is evident from the fact that the peak energy of the smaller amplitude solitary waves in the third and fourth block are only around 10 and 3 % respectively, of the maximum kinetic energy in the top block. For comparison, the kinetic energy in each block has been normalized to the peak value of the kinetic energy of the solitary wave in the top block. The speed of the largest amplitude solitary wave in each block with maximum compressive force also shows the same scaling as is observed for one-dimensional systems [39].

Completely developed solitary waves are not observed in systems with smaller block sizes, however, we do observe similar dynamics and comparable impact decimation ability as seen in larger sized systems. In the semilog plot in Fig. 3 we show the percentage of output kinetic energy for systems with different number of blocks N_B, and a range of restitution values (where $\varepsilon = 1,\ 0.99,\ 0.95$ and 0.9). The number of spheres in a single column of any block is taken

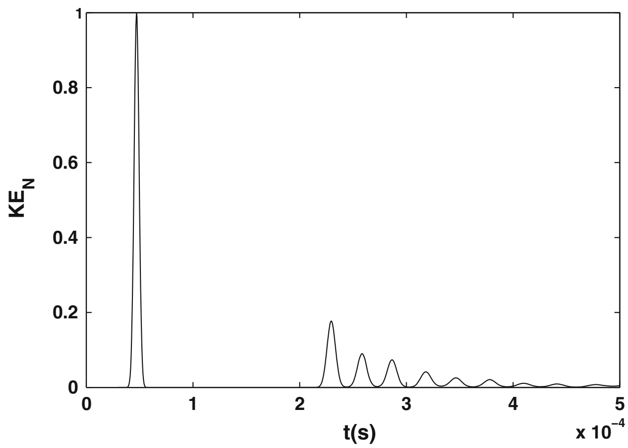

Fig. 4 Normalized kinetic energy for a 2-block system with top block made of steel spheres and the lower block made from Teflon spheres

to be 2^3. The output kinetic energy decreases significantly with the number of blocks. In Fig. 3 this decrease is seen to be exponential and the output energy for 1, 2, 3 and 4 block systems are approximately 34, 11, 4 and 1 %, respectively, of the incident energy for completely elastic systems. Inclusion of restitution further decreases the output kinetic energy with hardly any significant output for the 4 block system. The output kinetic energy, however, is still seen to decrease exponentially and for the range of restitution values considered can be approximated by $E_{out} \propto E_{in} e^{-(\alpha/\epsilon)N_B}$, with α a positive constant ($\alpha \approx 1.12$).

The dependence of the final velocity of the larger sphere on mass ratio κ in Eq. (3) can be further exploited to design panels at small length scales with higher impact decimation capability. Smaller κ values would lead to the larger sphere retaining most its energy at each collision. Since the mass ratio between the two spheres depends both on the radius and the density, we can envision a stacked panel arrangement where a block with lower density material is placed below a block with higher density material. In Fig. 4 we show the kinetic energy propagation for a two block system where the upper block is made from steel spheres and the lower block from Teflon spheres ($\rho = 2170$ kg/m^3). Compared to Fig. 2 the single solitary wave is seen to split into many more smaller solitary waves. The peak kinetic energy of the smaller amplitude solitary wave is almost half of that in Fig. 2 and the output kinetic energy is observed to be around 4 % of the incident kinetic energy. In addition, the speed of solitary waves decreases significantly in the lower block due to the low Young's modulus (Y = 1.46 GPa) of Teflon [39]. We can then think of a Teflon block sandwiched between blocks made from higher density steel material. The lowest steel block could then be used to split and speed up the outcoming energy from the Teflon block. In a sandwiched 3-block system, with a block made from Teflon surrounded on either side with blocks consisting of steel spheres we observed that the output kinetic energy was around 0.6 % of the input energy.

4 Conclusion

In conclusion, we have proposed a simple packing which shows the ability to successfully disperse impact energies. The system possesses two essential properties which are responsible for the observed impact decimation. Firstly, the size mismatch between the spheres in the two stacked blocks result in breaking of impact energy in smaller packets, and, secondly, the prong-like structure which leads to spreading of the energy into more columns in subsequent blocks. Impact mitigation through a 2D Y-shaped branched network of granular chains has been studied recently [40]. In this system for symmetric branching, the ratio of the wave amplitude in one of the branches to the amplitude of the incident wave is

$$T_S = \left(\frac{2\cos\alpha}{2\cos^2\alpha + 1}\right)^{6/5}, \tag{4}$$

where, α is the branch angle. For the proposed system following a similar approach it can be shown that this ratio is

$$T_S = \frac{1}{4}\left(\frac{2\cos\theta}{1 + 2\kappa\cos^2\theta}\right)^{6/5}. \tag{5}$$

If $\alpha = \theta$ then the ratios are 0.63 and 0.42 respectively, which shows the enhanced impact mitigation obtained by introducing a mass-mismatch. As has been shown, introducing a stacked block made from lighter material between blocks made from denser material further enhances the impact dispersion property of the proposed structure.

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