PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 113, Number 2, October 1991

UNKNOTTING NUMBER ONE KNOTS ARE PRIME: A NEW PROOF

XINGRU ZHANG

(Communicated by Frederick R. Cohen)

ABSTRACT. An alternative proof for unknotting number one knots being prime is given.

The unknotting number of a knot $K \subset S^3$, denoted by $u(K)$, is the minimum number of crossing changes required to unknot K. Obviously $u(K)$ is a knot invariant, but surprizingly little is known about it. The following theorem of M. Scharlemann proves a long standing conjecture.

Theorem [S, Theorem]. *A knot $K \subset S^3$ with $u(K) = 1$ is prime.*

To prove the above theorem, Scharlemann developed certain combinatorics dealing with planar graphs coming from an intersection of two special planar surfaces. Later in [ST], Scharlemann and Thompson gave another proof of the theorem [ST, Corollary 3.4], that is based on a delicate application of the sutured manifold structure theory. In this note we point out a new proof, applying only some existing results. In fact the proof follows immediately from the following three known lemmas.

Lemma 1 [L, Lemma 1]. *Let K be a knot in S^3 with $u(K) = 1$, and let M_K be the double cover of S^3 branched over K. Then M_K can be obtained by $n/2$-surgery on some knot in S^3, n being an odd integer.*

Lemma 2 [GL, Theorem 1]. *Let K be a knot in S^3, and let $K(m/l)$ denote the manifold obtained by m/l-surgery on K. Then $K(m/l)$ is a prime manifold if $|l| \neq 1$.*

Lemma 3 [KT, Corollary 4]. *Let K be a knot in S^3. Then the double cover M_K of S^3 branched over K is a prime manifold iff K is a prime knot.*

Proof of Theorem. Since $u(K) = 1$, M_K is a prime manifold by Lemmas 1 and 2. Hence K is a prime knot by Lemma 3.

The author has learned that a similar approach was pointed out by C. Gordon in a lecture given at Santa Barbara.

Received by the editors June 5, 1990.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 57M25.

References

[GL] C. McA. Gordon and J. Luecke, *Only integral Dehn surgery can yield reducible manifolds*, Math. Proc. Cambridge Philos. Soc. **102** (1987), 97–101.

[KT] P. K. Kim and J. L. Tollefson, *Splitting the PL involutions of nonprime* 3*-manifolds*, Michigan Math. J. **27** (1980), 259–274.

[L] W. B. R. Lickorish, *The unknotting number of a classical knot*, Contemp. Math., vol. 44, 1985, pp. 117–121.

[S] M. G. Scharlemann, *Unknotting number one knots are prime*, Invent. Math. **82** (1985), 37–55.

[ST] M. G. Scharlemann and A. Thompson, *Unknotting number, genus, and companion tori*, Math. Ann. **280** (1988), 191–205.

Mathematics Department, University of British Columbia, Vancouver, British Columbia, Canada, V6T 1Y4