Math 417-517 - Fall 2018 - Problems and Announcements

(Problems in Greenberg: Advanced Engineering Mathematics)

1. (Aug 28) review chapters 9 and 10

2. (Aug 30) section 13.3, problems 2(a), 3(a,f).
   Also special problem: Find the tangent plane to \( z = 1/(x^2 + y^2) \) at \( x = 1, y = 2 \). (Answer: \( z = \frac{3}{5} - \frac{2}{25}x - \frac{4}{25}y \).)

3. (Sept 4) section 13.4, problems 3a, 4a, 5a, 6(a,d).

4. (Sept 6) section 13.6, problems 2a, 3a, 4a, 5a, 6a, 12.

5. (Sept 11) special problem: The function \( x = u^3 - v^3, y = (u + v)^2 \) sends \( (1,0) \) to \( (1,1) \). Show there is an inverse function \( u = u(x,y), v = v(x,y) \) defined near \( (1,1) \) and find all the partial derivatives at that point.
   (Answer: \( u_x = \frac{1}{3}, u_y = 0, v_x = -\frac{1}{3}, v_y = \frac{1}{2} \).)

6. (Sept 13) section 13.7 problems 5(a,g), 10.

7. (Sept 18) section 13.8 problems 1(a,d), 3, 6.

8. (Sept 20) Three problems on calculus of variations:
   
   (a) Find the function \( y = y(x) \) with \( y(0) = 0, y(1) = 0 \) which minimizes the integral
   
   \[
   I(y) = \int_0^1 ((y')^2 + 12xy)dx
   \]

   What is the minimum value of \( I(y) \)? (Ans: \( y = x^3 - x, I(y) = -\frac{4}{5} \))

   (b) A pendulum of mass \( m \) on a string of length \( \ell \) makes an angle \( \theta = \theta(t) \) with the verticle at time \( t \). The kinetic energy is \( T = \frac{1}{2}ml^2\theta'^2 \), the potential energy is \( V = mgl(1 - \cos \theta) \), and the Lagrangian is the difference \( L = T - V \). To find the motion we seek to minimize the action \( I(\theta) = \int_{t_0}^{t_1} L(\theta, \theta')dt \). What is the equation of motion we get from this formulation?
(c) Show that the Euler equation can be written in the form

$$\frac{d}{dx}(F - y'F_y') = F_x$$

and conclude that if $F(x, y, y')$ does not explicitly depend on $x$ then $F - y'F_y' = \text{constant}$.

9. (Sept 25)
   section 14.3 problems 1(a,h,k)
   section 14.4 problems 4(a,d)
   section 14.5 problems 1(a,d), 7(a)

10. (Sept 27)
    section 14.6 problems 3b, 5d, 9c

11. (Oct 9)
    section 15.2 problems 1f, 2a, 3a

12. (Oct 11)
    section 15.3 problems 1a, 4a, 12(a,c)

13. (Oct 16)
    section 15.4 problems 7, 11(a,g)
    section 15.5 problems 1(a,c), 10(a,c,e)

14. (Oct 18)
    section 15.5 problems 13(a,d)
    section 15.6 problems 2, 4a

15. (Oct 23)
    section 16.4 problems 2d, 4a
    section 16.5 problems 1(a,d)
    section 16.6 problems 1d, 6f
16. (Oct 25)

section 16.8 problems 1(d,l).

Also special problem: Verify the divergence theorem for the sphere of radius $a$ and the vector field $\mathbf{v} = \mathbf{R}|\mathbf{R}|^2$.

17. (Oct 30) section 16.9, problems 2e, 3(d,g), 4

18. (Nov 1) section 16.10, problems 2(f,k), 5(a,c)

19. (Nov 6) special problem: consider the coordinate system

$$x = \cosh u_1 \cos u_2 \quad y = \sinh u_1 \sin u_2 \quad z = u_3$$

Find the metric coefficients $g_{ij}$, the element of arc length $ds$, and the volume element $dV$.

20. (Nov 8) section 16.7, problems 2d, 6, 11d.

21. (Nov 13) section 21.2 problems 9(a,e), 10(a,b)

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**First exam:** The first exam was on Thursday, October 4. It covered Chapter 13 (except 13.5), the calculus of variations, and Chapter 14. No laptops, calculators, note cards, scratch paper allowed. Only a pen or pencil.

**Second exam:** The second exam is on Thursday, November 15. It will cover chapters 15 and 16. No laptops, calculators, note cards, scratch paper allowed. Only a pen or pencil. (Final exam is December 13)