

## FINAL EXAM REVIEW SHEET

You should prepare at least the following topics

No list can cover everything, but these are the main things

Basic integration rules

$$\int x^n dx = x^{n+1}/(n+1) + C, \text{ if } n \text{ not equal to } -1$$

$$\int x^{-1} dx = \ln x + C$$

$$\int e^x dx = e^x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \tan x dx = \ln |\sec x| + C$$

$$\int \sec x dx = \ln |\sec x + \tan x| + C$$

Don't forget the "+ C" in indefinite integrals

Go over chain rule for derivatives

Review rules for exponents and roots

Substitution

$$\int f(u) du/dx dx = \int f(u) du$$

Guess and check backwards--usually much faster

Some problems can only be done by substitution

E.g.  $\int x\sqrt{x+1} dx$

Substitution in definite integrals

Either change the limits to  $u = c$ ,  $u = d$  or plug in  $x$  limits at the end

Don't put a "+ C" in a definite integral

Area between two curves

Be sure to check which curve is on top/right

Sometimes the curves switch from being on top to being on bottom  
or from being on right to being on left

Volumes

Slicing

Discs/washers

Cylindrical shells

Be sure to know when you use which

Integration Methods

Integration by Parts

Trig integrals

Odd powers of sin or cos

Even powers of sin or cos (use only if no odd powers around)

Odd power of tan

Even power of sec

Odd power of sec (use Integration by Parts)

Trig substitution

BEWARE the simple substitution trap

Always check for u-substitution before doing trig substitution

Using the right triangle to convert back to x (figures 3 & 4 on p 492)

Completing the square

Partial Fractions

BEWARE the partial fraction trap

Partial fractions puts an integral into a certain form

If it is already in that form, partial fractions won't help

Rationalizing substitutions

Sometimes works to let  $u =$  the n-th root of some expression

Strategy for integration

Check for substitution

Expand out, or substitute and expand out

For products or inverse functions, try IBP

For functions with square roots of  $x^2$ , or other expressions in  $x^2$   
use trig sub

For rational functions, use partial fractions

LOOK BEFORE YOU INTEGRATE

Numerical integration (approximation of definite integrals)

Trapezoidal rule

Simpson's rule

On an exam, these are usually 'set up problems'

Leave answer in a form that could be evaluated with a calculator

Improper Integrals

Integrating to infinity

Function goes to infinity at an end point

Function goes to infinity at an interior point

Must use limits to get credit

Convergence/divergence of improper integrals

p-integral at 0; at infinity

Comparison test

Limit comparison test

Arc length/surface area, including parametric curves

$$ds = \sqrt{dx^2 + dy^2}$$

Be sure you know how to interpret this formula  
These problems are very unforgiving; be careful

Law of exponential growth

$$y' = ky \text{ implies } y = e^{kt}$$

Growth of  $y$  proportional to  $y$  implies  $y = C b^t$

Spread of disease, growth of bacteria, etc.

Newton's Law of cooling

$$T - T_s = (T_0 - T_s)b^t$$

Sequences

Limits

Convergence/divergence

Recursion

Limit Theorems

Sum

Constant multiple

Product

Quotient

Squeeze

Series

Definition of series equals  $L$

Partial sums approach  $L$

What is a partial sum?

Geometric series

When does it converge? Diverge?

What if it doesn't start with 1?

Test for series

n-th term divergence test

If terms don't go to 0, then series diverges

A series can diverge even if the terms go to 0

"Terms go to 0" is not an answer to "Why does the series converge?"

Harmonic series  $1 + 1/2 + 1/3 + 1/4 + \dots$

Integral Test

Most useful for p-series, logarithmic p-series

Comparison Test

Limit comparison--most useful

Ratio test

Works best with powers and factorials

Go over factorials

Go over powers

Leibnitz Test

Alternating harmonic series

Absolute convergence/conditional convergence/divergence

Power series

Radius of convergence

Put absolute values on everything & use ratio test

End points

Don't use ratio test; inconclusive by definition at the end points

Use Leibnitz test or limit comparison

Interval of convergence

Differentiation/integration term by term

Power series obtained by differentiating/integrating geometric series

Taylor Series

Definition of Taylor Series

Taylor series of familiar functions

$1/(1+x)$ ,  $1/(1-x)$ ,  $1/(1+x^2)$ ,  $e^x$ ,  $\sin x$ ,  $\cos x$

If a power series converges to a function it *is* the Taylor Series

Using familiar Taylor Series to obtain new ones

E.g. Find Taylor Series of  $x^4 e^{5x}$