MTH 411/511 **PROBABILITY AXIOMS** Prof. Cowen

A *probability* or a *probability measure* P is a function from the events in a sample space Ω to the real numbers such that

Axiom 1: $0 \leq P(E) \leq 1$ for each event E in Ω.

Axiom 2: $P(\Omega) = 1$.

Axiom 3:

i) **Additivity** If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$

ii) **Finite additivity** If $E_1, ...E_n$ are mutually exclusive events then
$$P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i)$$

iii) **Countable additivity** If $E_1, E_2, E_3, \ldots$ are mutually exclusive events then
$$P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$$

Note: the sum in iii) always converges.

CONSEQUENCES OF AXIOMS

1) If P is a probability on a sample space Ω, then $P(\varnothing) = 0$.

2) If E is an event then $P(E^c) = 1 - P(E)$.

3) If E and F are events and $E \subset F$, then $P(F-E) = P(F) - P(E)$

4) If E and F are events and $E \subset F$, then $P(E) \leq P(F)$.

5) If E and F are events, then $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

6) If E and F are events, then $P(F) = P(E \cap F) + P(E^c \cap F)$

7) If E, F, and G are events, then $P(E \cup F \cup G) = P(E) + P(F) + P(G)$
$- P(E \cap F) - P(F \cap G) - P(G \cap E) + P(E \cap F \cap G)$

8) If $E_1 \subset E_2 \subset E_3 \subset \ldots$ are events, then $P(\bigcup_{i=1}^{\infty} E_i) = \lim_{n \to \infty} P(E_n)$

9) If $E_1 \supset E_2 \supset E_3 \supset \ldots$ are events, then $P(\bigcap_{i=1}^{\infty} E_i) = \lim_{n \to \infty} P(E_n)$