MTH 411/511

Prof. Cowen **Review of Sets**

Definition 1 (Subset)**.** Let A and B be sets. We say A is *contained in* B or A is a *subset* of B, written $A \subset B$, if whenever $x \in A$, then $x \in B$. As an alternative, we sometimes say that B *contains* A, written $B \supset A$.

Note: some people write $A \subseteq B$ (which is consistent with writing $x \leq y$) instead of $A \subset B$. You may use whichever you would like, but understand that writing $A \subset B$ means that A could possibly equal B (unlike when writing $x < y$).

Definition 2 (Equality of Sets)**.** We say that A *equals* B, written $A = B$, if $A \subset B$ and $B \subset A$.

Definition 3 (Union and intersection)**.** The *union* $A \cup B$ of two sets A and B is defined by $A \cup B := \{x \mid x \in A \text{ or } x \in B\}$. Note that 'or' in mathematics means 'and/or.' The *intersection* $A \cap B$ is defined by $A \cap B := \{x \mid x \in A \text{ and } x \in B\}$.

Note: Our textbook use AB as shorthand for $A \cap B$.

Definition 4 (Union and intersection of n sets)**.** Let $A_1, \ldots, A_n$ be sets. The *union* $\bigcup_{i=1}^n A_i$ of $A_1, \ldots, A_n$ is defined by $\bigcup_{i=1}^n A_i := \{x \mid x \in A_i \text{ for at least one } i,\ 1 \leq i \leq n\}$. The *intersection* $\bigcap_{i=1}^n A_i$ of $A_1, \ldots, A_n$ is defined by $\bigcap_{i=1}^n A_i := \{x \mid x \in A_i \text{ for all } i,\ 1 \leq i \leq n\}$.

Definition 5 (Union and intersection of infinitely many sets)**.** Let $A_1, A_2, A_3, \ldots$ be sets. The *union* $\bigcup_{i=1}^\infty A_i$ of $A_1, A_2, A_3, \ldots$ is defined by $\bigcup_{i=1}^\infty A_i := \{x \mid x \in A_i \text{ for at least one } i \geq 1\}$. The *intersection* $\bigcap_{i=1}^\infty A_i$ of $A_1, A_2, A_3, \ldots$ is defined by $\bigcap_{i=1}^\infty A_i := \{x \mid x \in A_i \text{ for all } i \geq 1\}$.

Definition 6 (Empty Set)**.** The *empty set* $\varnothing$ is the set with no elements.

Definition 7 (Disjoint or mutually exclusive)**.** Sets A and B are *disjoint* or *mutually exclusive* if $A \cap B = \varnothing$.

Definition 8 (Mutually exclusive)**.** Sets $A_1, A_2, \ldots A_n$ or $A_1, A_2, A_3, \ldots$ are *mutually exclusive* if $A_i \cap A_j = \varnothing$ for all $i \neq j$.

Note: for more than 2 sets, mutually exclusive is the same as *pairwise disjoint.*

Proposition 9 (Basic properties of sets)**.** *If A, B, C are sets, then*

(1) $\varnothing \cap A = \varnothing$; $\varnothing \cup A = A$

(2) $A \cap B \subset A$

(3) $A \subset A \cup B$

(4) $A \cup B = B \cup A$; $A \cap B = B \cap A$

(5) $A \cup (B \cup C) = A \cup B \cup C = (A \cup B) \cup C$

(6) $A \cup A = A \cap A = A$

(7) If $A \subset B$, then $A \cup C \subset B \cup C$ and $A \cap C \subset B \cap C$

Proposition 10 (Distributive Rules)**.** *If A, B, C are sets, then*

(1) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(2) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Definition 11 (Relative complement or difference)**.** If X and A are sets, the *relative complement* $X - A$ of A in X is defined by $X - A := \{x \in X \mid x \notin A\}$. Sometimes this is called the *difference* of X and A.

Proposition 12 (de Morgan's laws)**.** *If A, B and X are sets then*

(1) $X - (A \cup B) = (X - A) \cap (X - B)$

(2) $X - (A \cap B) = (X - A) \cup (X - B)$

In probability, we often deal with sets that are all subsets of one set Ω.

Definition 13 (Complement)**.** If A is a subset of Ω, then the *complement* A^c of A is defined by $A^c := \{x \in \Omega \mid x \notin A\}$.

Proposition 14 (de Morgan's laws)**.** *If A and B are subsets of Ω then*

(1) $(A \cup B)^c = A^c \cap B^c$

(2) $(A \cap B)^c = A^c \cup B^c$

Proposition 15 (de Morgan's laws for n sets or infinitely many sets)**.** *If $A_1, A_2, \ldots A_n$ or $A_1, A_2, A_3, \ldots$ are subsets of Ω then*

(1) $(\bigcup_{i=1} A_i)^c = \bigcap_{i=1} A_i^c$

(2) $(\bigcap_{i=1} A_i)^c = \bigcup_{i=1} A_i^c$