MTH 460/560 Problem 22 page 249 Spring 08

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Using the notation of pages 238 and 239, we have $A\tilde{q}$ is in both G and H, where $G = \{Aq \mid q \in Q\}$ and $H = \{x \in \mathbf{R}^5 \mid x \leq ve\}$. Since in our problem the intersection of G with H consists of only one point, namely $(3,3)^t$, then $A\tilde{q} = (3,3)^t$.

Let $\tilde{q} = (q_1, \ldots, q_5)^t$. Then we have:

$$1q_1 + 2q_2 + 3q_3 + 4q_4 + 5q_5 = 3 \text{ and} \tag{1}$$

$$5q_1 + 4q_2 + 3q_3 + 2q_4 + 1q_5 = 3 \tag{2}$$

Subtracting the first equation from the second gives:

$$4q_1 + 2q_2 - 2q_4 - 4q_5 = 0.$$

Dividing by 2 gives that

$$2q_1 + q_2 = q_4 + 2q_5.$$

(so what I said in class on Monday about $q_1 = q_5$ and $q_2 = q_4$ is not the whole the story).

So the answer is: The set of all security strategies $\tilde{q}$ for Player 2 is the set of all $(q_1, ..., q_5)^t$ such that $q_1 + ... + q_5 = 1$, $q_i \geq 0$ for $i = 1, \ldots, 5$, and $2q_1 + q_2 = q_4 + 2q_5$.

For example, $q = (0.1, 0.3, 0.15, 0.4, 0.05)^t$ is a security strategy for Player 2. (You can check that if Player 2 uses this strategy then Player 1's payoff matrix is $(3,3)^t$, so no matter how he mixes he cannot do better than 3, the value of the game.