MTH 539 – Methods of Applied Mathematics **Fall 2018**

Instructor: Brian Spencer tel: 645-8805
office: 319 Mathematics Bldg email: spencerb@buffalo.edu

Lectures: Tue/Thu 8:00 – 9:20 in Math 250
Office Hours:

Course Information

Prerequisites: undergraduate linear algebra and undergraduate differential equations.

Description: This two-part course (MTH539, MTH540) is for graduate students in mathematics, science, and engineering who wish to learn advanced mathematical methods for solving linear systems, differential equations, integral equations and partial differential equations, as well as learn the basics of calculus of variations, transform theory, asymptotic and perturbation methods, and bifurcation theory.

MTH 539 (Part 1) describes a unified set of analytical techniques for constructing solutions to linear algebraic systems, differential equations and integral equations. Also included is an introduction to the calculus of variations. MTH 539 does not require knowledge of partial differential equations.

1. **Vector Spaces and Linear Systems**
   - Linear vector spaces and representation in terms of basis vectors
   - spectral theory for matrices, adjoint matrices and properties of eigenvalue problem
   - Fredholm alternative theorem, least squares solutions, singular value decomposition
2. **Function Spaces**
   - function space, continuity, differentiability, inner product, L^2 norm, Hilbert space
   - representation of functions: Fourier series, orthogonal polynomials, finite elements
3. **Integral Equations**
   - classification of integral equations (Hilbert-Schmidt, Volterra, Fredholm, first-kind, second-kind)
   - solution methods for integral equations
   - domain, range, adjoint and Fredholm alternative for linear operator
   - spectral theory for integral operators and correspondence with matrix operators
4. **Differential equations and Green's functions**
   - delta functions, Green's functions, distribution theory and weak solutions
   - construction of Green's functions for boundary value problems and initial value problems
   - spectral theory of differential operators, adjoint, Fredholm alternative; correspondence with integral operators
   - Sturm-Liouville boundary value problems
   - solution to boundary value problems using eigenfunction expansions (Fourier, Legendre, Chebyshev, Bessel)
5. **Calculus of variations**
   - minimization of functionals and Euler-Lagrange equations
   - Hamilton's principle
   - minimization of functions and relation to eigenvalues of Sturm-Liouville operators

MTH 540 (Part 2) includes an introduction to transform theory, methods for solving partial differential equations, and a comprehensive study of asymptotic methods as a tool for finding useful approximate solutions to differential equations. MTH 540 requires MTH 539 and some familiarity with partial differential equations. A detailed list of topics can be found at `http://www.nsm.buffalo.edu/~spencerb/detailed_syllabus_539-540.pdf`

Course Materials

Primary Text: James P. Keener, *Principles of Applied Mathematics*
[Westview Press; updated, revised version (2000); ISBN-10: 0738201294 ⟸ Note correct ISBN]
Available through Amazon for $93 and on reserve in Lockwood Library (2-hour in-library loan).

Alternate Texts for MTH 539:

- Francis B. Hildebrandt, *Methods of Applied Mathematics* (on reserve in Lockwood Library)
- Bernard Friedman, *Principles and Techniques of Applied Mathematics (full text online through UB Library)*
- Ivar Stakgold, *Boundary Value Problems of Mathematical Physics*, Vol 1 (on reserve in Lockwood Library)

UBLearns: contains homework assignments, announcements, MATLAB programs and occasional copies of lecture notes/handouts.

Computational Software: For some topics, example calculations will be done with the computational software MATLAB. Students will be required to use MATLAB (or Python or Mathematica or Maple if preferred) for some assignments. Examples of such calculations in MATLAB will be presented in lecture, with a copy of the program available for student download via UBLearns. MATLAB is available on campus computers (Cybrary or Linux labs) and for download to personal computers as detailed here:
`http://www.buffalo.edu/ubit/service-guides/software.html`

Coursework

Homework – Almost every class there will be a homework assignment. Homework is usually collected weekly. The homework grade is determined by adding the total points of all assignments and using a grading scale similar to the rubric below.

Midterm Exam – In-class exam on chapters 1-2 on Tuesday October 9. The exam grade will be determined by adding the total points on the exam and using a grading scale similar to the rubric below.

Final Exam - Cumulative final exam during exam week. Currently scheduled for 8-11am Tuesday December 11 in Math 250. See MyUB to confirm schedule. The final exam must be taken during its scheduled time.

Grades

Course Grades are determined by averaging the grades with the following weightings:

Homework	35%
Midterm Exam	20%
Final Exam	45%

Grading Rubric:
Points for homework and exam questions are based on the following guidelines (assuming a 10 point question):

10/10 - Correct method, clearly presented, correct answer. No significant mistakes (grade A work).

8/10 - Correctly captures the essential method or idea in the solution of the problem, is clearly presented, but has one or more minor errors (grade B work).

6/10 - Displays some understanding of the underlying concepts and ideas but the solution contains significant errors in execution of the details (grade C work).

4/10 - Questionable understanding of the underlying concepts and ideas and/or major errors (grade D work).

2/10 - Minimal progress, but some parts of the solution are not totally incorrect (grade F work).

0/10 - The solution has no redeeming features (grade F).

Point scores are scaled proportionally for problems of 5 points, 15 points, etc.

5-point grading scale: For keeping track of course grades on exams and homework a 5-point scale is used:

A+ = 4.66-5.00	B+ = 3.66-4.00	C+ = 2.66-3.00	D+ = 1.66-2.00	F = 0 - 1.00
A = 4.33-4.66	B = 3.33-3.66	C = 2.33-2.66	D = 1.33-1.66	
A- = 4.00-4.33	B- = 3.00-3.33	C- = 2.00-2.33	D- = 1.00-1.33	

+/- grades will be used in assigning course grades. Note the university does not permit A+, C-, D+ or D- course grades for graduate students (A+ is submitted as A, C- as C, D+ or D- as D).

Late homework policy: Late homework is accepted with the following penalties:
- turned in by 5pm on due date: -10%
- turned in by 5pm day after due date: -20%
- 20% penalty per school day for each day thereafter.

Attendance: Class attendance is expected. Since the 8am timeslot is not user-friendly for many students, as an added incentive for attending, I occasionally take attendance and each class counts 1 point towards the homework grade (15-20 points over the semester). If you do miss a class you are responsible for getting the homework, lecture notes and any other in-class information or materials from a classmate.

Academic Honesty: Students must obey the university policy on academic integrity. Cheating, plagiarism, or representing the work of others as your own will be formally sanctioned following the university policies on Academic Integrity as outlined at http://grad.buffalo.edu/succeed/current-students/policy-library.html

Incompletes: Incompletes will be given only under extraordinary circumstances (like surgery during the last week of class).

Student Learning Outcomes

The MTH539-540 sequence covers fundamental methods and concepts in solving problems in applied mathematics. These topics are important as part of the broad mathematics background needed for research in graduate applied mathematics and other fields.

Upon successful completion of the course MTH 539 the student will be able to

- Solve linear algebra systems using one or more of the following as applicable: eigenvalues, spectral theory, Fredholm alternative theorem for existence and uniqueness of solutions, least squares solutions to overdetermined and underdetermined systems, pseudo-inverses.
- Represent functions using approximation methods: Fourier series, orthogonal polynomials, discrete Fourier transforms, finite elements.
- Classify and solve linear and singular integral equations using resolvent kernels, series solutions, discretized solutions.
- Solve linear differential equations (boundary value problems and initial value problems) using Green's functions and eigenfunction expansions. Use adjoint operators and Fredholm solvability theory for existence and uniqueness of solutions, least squares solutions.
- Use calculus of variations to minimize an energy functional with respect to a function. Includes derivation of the Euler-Lagrange equations for energy minimization and generalizations, as well as Hamilton's principle as applied to time-dependent systems.

Other

Important Dates:
Tue Sep 4 - Last day to drop the course - no record appears on transcript.
Fri Nov 9 - Last day to resign from the course - an 'R' appears on transcript.

Students with disabilities:
If you have a diagnosed disability (physical, learning, or psychological) which will make it difficult for you to carry out the course work as outlined, or requires accommodations such as recruiting note takers, readers, or extended time on exams and/or assignments, please advise me during the first two weeks of the course so that we may review possible arrangements for reasonable accommodations.

Lecture plan

tue 8/28 - syllabus, course intro

thu 8/30- intro to linear vector spaces [1.1 of Keener book]

tue 9/4 - orthogonal vectors, basis vectors and Gram-Schmidt orthogonalization [1.1]; matlab demo

thu 9/6 - spectral theory for matrices, solution to linear problems by spectral decomposition [1.2]

tue 9/11 - geometric interpretation of eigenvalues [1.3]; Fredholm alternative theorem for linear systems [1.4]

thu 9/13 - least squares solutions and pseudoinverses [1.5]

tue 9/18 - methods for solving linear systems: gaussian elimination, singular value decomposition [1.5]

thu 9/20 - intro to function spaces and representation of functions [2.1, 2.2]

tue 9/25 - orthogonal polynomials [2.2]

thu 9/27 - trigonometric series [2.2]

tue 10/2 - finite elements [2.2]

thu 10/4 - review for midterm

tue 10/9 - midterm exam

thu 10/11 - intro to integral equations [3.1]; bounded linear operators and Fredholm alternative [3.2]

tue 10/16 - compact operators [3.3]; spectral theory for integral operators [3.4]

thu 10/18 - resolvent and pseudo-resolvent kernels [3.5]; approximate solutions to integral equations [3.6]

tue 10/23 - distributions and the delta function; weak solutions [4.1]

thu 10/25 - Green's functions and construction of Green's function [4.2]

tue 10/30 - more Green's function examples [4.2]

thu 11/1 - adjoint operator, adjoint Green's function [4.3]

tue 11/6 - Fredholm alternative for linear differential operators [4.3]

thu 11/8 - least squares solutions, modified Green's functions [4.4]

tue 11/13 - spectral theory of differential operators, Sturm-Liouville boundary value problems [4.5]

thu 11/15 - solution to BVP using eigenfunction expansions (Fourier, Legendre, Chebyshev, Bessel) [4.5]

tue 11/20 - introduction to calculus of variations, Euler-Lagrange equations, constraints [5.1]

thu 11/22 - no class (fall recess)

tue 11/27 - more examples of Euler-Lagrange [5.1]; Hamiltonian [5.2]

thu 11/29 - Hamilton's equations with examples for pendulum, vibrating string [5.2]

tue 12/4 - minimization of functions and relation to eigenvalues of Sturm-Liouville operators [5.3, 5.4]

thu 12/6 - review for final exam

tue 12/11 - Final Exam in Math Bldg 250 from 8:00am – 11:00am (check MyUB to confirm)